GAS DYNAMICS OF COSMIC CLOUDS

INTERNATIONAL UNION
OF THEORETICAL AND APPLIED MECHANICS AND
INTERNATIONAL ASTRONOMICAL UNION

GAS DYNAMICS OF COSMIC CLOUDS

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PREFACE

The Second Symposium on Cosmical Gas Dynamics was organised by cooperation between the International Astronomical Union and the International Union of Theoretical and Applied Mechanics, as a sequel to the Symposium arranged by the same Unions in August 1949 at Paris.*

The Organising Committee consisted of the following scientists:

for IUTAM: Th. von Kármán, Pasadena, Calif.; G. I. Taylor, Cambridge, England; A. R. Kantrowitz, Ithaca, N.Y.; J. M. Burgers, Delft, Holland;

for IAU: J. H. Oort, Leiden, Holland; H. C. van de Hulst, Leiden, Holland; F. Hoyle, Cambridge, England; L. Spitzer Jr., Princeton, N. J.

Each of the two Unions asked for a grant-in-aid from UNESCO and through UNESCO's valuable help a sum of \$ 6000 was received, the same amount as had been given on behalf of the first Symposium, in order to cover travelling expenses of scientists coming to the Symposium. A further sum of \$ 1000 was received as assistance for publishing the Proceedings of the Symposium.

Cambridge in England was selected as a suitable place of meeting, and the dates were fixed at July 6–11, 1954.

Dr. G. K. Batchelor offered to take care of the local arrangements. Two rooms in the Engineering Laboratory of the University of Cambridge were made available. One served for the lectures, while the other one was used as a common room for informal discussions, the exhibition of sets of stellar photographs and for coffee and tea. The expenses of these items were carried by the Unions.

The aims of the Symposium were similar to those of the preceding one: to bring together workers from astrophysics and from aerodynamics; to give an opportunity to astronomers to expound the present state of their problems and to make aerodynamicists acquainted with the observational facts; to consider which developments in fluid mechanics

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* A small stock of copies of the Proceedings of the Paris Symposium published by the Central Air Documents Office, Army-Navy-Air Force, Dayton, Ohio, U.S.A., under the title "Problems of Cosmical Aerodynamics", is still available. Applications should be directed to Col. C. E. Davies, Secretary U.S. National Committee on Theoretical and Applied Mechanics, Room 1101, 29 West 39th street, New York 18, N.Y., U.S.A.; the price is \$0,75 per copy for administration expenses.

may be applicable to astrophysical problems, and to arrive at a formulation of these problems in such a way that mathematicians and fluid mechanics people may find a way of attack.

After preliminary discussions and correspondence during 1952, the Organising Committee made up a draft programme, which in December of that year was forwarded to a number of scientists. During the first months of 1953 the programme underwent some adjustment, but care was taken to leave a certain freedom for arrangements to be made at the meeting itself.

It was considered helpful for the discussions at the Symposium to ask several prospective participants to prepare summaries of their ideas on certain subjects, which could be mimeographed and circularised beforehand. This has proved to be of great help, although a few of the summaries could be distributed only at the beginning of the Symposium. The titles of these documents were as follows:

No. 1: H. Zanstra (Amsterdam), On the Formation of Condensations in a Gaseous Nebula and in Interstellar Matter, 20 pp. + 3 diagrams (with an additional "Remark").
No. 2: A. D. Thackeray (Pretoria), Characteristics of some Diffuse Nebulae in the Magellanic Clouds, 2 pp.
No. 3: F. D. Kahn (Manchester), The Heating and Cooling of Interstellar Gas Clouds, 13 pp.
No. 4: M. P. Savedoff (Leiden), Relation between Dust and Gas, 2 pp.
No. 5: S. Chandrasekhar and E. Fermi (Chicago), Magnetic Fields in Spiral Arms, 5 pp.
No. 6: B. J. Bok and C. M. Wade (Harvard), A Preliminary Classification System for Hα-Emission Nebulae, 2 pp.
No. 7: D. Hoffleit (Harvard), Observational Features in Carina of Interest for Dynamical Theories of Interstellar Clouds and Stellar Evolution, 5 pp.
No. 8: W. H. McCrea (Cambridge), Motion of Stars through Clouds; Accretion, 7 pp.
No. 9: H. C. van de Hulst (Leiden), List of Problems and Suggested Solutions, 3 pp.
No. 10: L. Biermann and A. Schlüter (Göttingen), On the Influence of Radiation, Ionization and Magnetic fields on the Dynamics of the Interstellar Medium, 6 pp.
No. 11: L. Biermann (Göttingen), On the Mass Balance of the Interstellar Medium, 4 pp.

No. 12: A. SCHLÜTER, H. SCHMIDT and P. STUMPFF (Göttingen and Bonn), An Analysis of the Interstellar Ca-lines, 3 pp.

No. 13: S. VON HOERNER (Göttingen), Contribution to the Turbulence Theory of Galaxies, 3 pp.

No. 14: M. P. SAVEDOFF (Leiden), The Energy Balance of the Interstellar Medium, 6 pp.

No. 15: J. H. OORT, (Leiden), Origin of Interstellar Clouds and Their Acceleration, 3 pp.

The following scientists took part in the meetings:

Belgium: P. Ledoux (Liège).

Finland: J. Tuominen (Helsinki).

France: J. F. Denisse (Paris); J. Kampé de Fériet (Lille); E. Schatzman (Paris).

Germany: F. Becker (Bonn); L. Biermann, S. von Hoerner, A. Schlüter, C. F. von Weizsäcker (all from Göttingen).

Great Britain:

(a) from Cambridge itself: E. W. Bastin, G. K. Batchelor, A. Beer, H. Bondi, R. Hide, F. Hoyle, H. Jeffreys, H. von Klüber, E. H. Linfoot, R. A. Lyttleton, W. H. McCrea, R. O. Redman, W. H. Reid, M. Ryle, D. W. Sciama, F. J. M. Stratton, G. I. Taylor;

(b) from other places in Great Britain:
E. C. Bullard (Teddington); T. G. Cowling (Leeds); T. Gold (Herstmonceux); R. C. Jennison (Jodrell Bank, Manchester); F. D. Kahn (Manchester); M. J. Lighthill (Manchester); A. Maxwell (Manchester); L. Mestel (Leeds); W. H. Ramsey (Manchester); M. J. Seaton (London); G. Temple (London).

India: M. K. Das Gupta (temporarily at Manchester).

Japan: Z. Suemoto (Tokyo, temporarily at Cambridge).

Netherlands: A. Blaauw (Leiden); L. J. F. Broer (Delft); J. M. Burgers (Delft); H. C. van de Hulst (Leiden); J. H. Oort (Leiden); H. Zanstra (Amsterdam);

United States of America: Bart J. Bok and Priscilla F. Bok (Cambridge, Mass.); A. Deutsch (Pasadena, Calif.); H. W. Emmons (Cambridge, Mass., temporarily at Cambridge, England); F. N. Frenkiel (Washington D. C.); W. D. Hayes (Providence, R. I., temporarily at London); E. P. Hubble (Pasadena, Calif.); A. R. Kantrowitz (Ithaca, N.Y.); Th. von Kármán (Pasadena, Calif., and Paris, France); O. Laporte (Ann Arbor, Mich.); H. W. Liepmann (Pasadena, Calif.); F. E. Marble (Pasadena, Calif.); D. H. Menzel (Cam-

bridge, Mass.); R. Minkowski (Pasadena, Calif.); M. P. Savedoff (Rochester, N.Y., temporarily at Leiden, Netherlands); S. A. Schaaf (Berkeley, Calif.); R. N. Thomas (Cambridge, Mass.).

Several other scientists had been invited, but unfortunately had no opportunity to be present at the Symposium.

Of the 63 participants 41 may be counted as astronomers or astrophysicists, 22 as aerodynamicists or physicists. The precise attribution to these categories is not always possible.

The arrangement of the sessions was as follows:

Monday, July 6, afternoon (chairman: Sir Geoffrey I. Taylor):
Astronomical data.

Tuesday, July 7, morning (chairman: Dr. E. C. Bullard):
Survey of problems; influence of electric conductivity; temperature regulation in the interstellar gas.

Tuesday, July 7, afternoon (chairman: Professor G. Temple):
Experimental investigation of shock waves; discussion on luminous edges of interstellar clouds.

Wednesday, July 8, morning (chairman: Dr. Th. von Kármán):
Effects of magnetic fields and of compressibility on turbulence.

Thursday, July 9, morning (chairman: Professor T. G. Cowling):
Origin of cosmic clouds and of their movement.

Friday, July 10, morning (chairman: Professor P. Ledoux):
Motion of stars through clouds and accretion.

Friday, July 10, afternoon (chairman: Professor J. H. Oort):
Mass balance of the interstellar medium.

Saturday, July 11, morning (chairman: Dr. H. Bondi):
Relation between gas and dust; summary of the Symposium and final discussion.

The full titles of the various papers with the names of the speakers and the subjects treated in the discussions are found in the "Table of Contents".

The Secretariate of the Symposium was in the hands of J. M. Burgers and H. C. van de Hulst. With the help of some others notes were taken of the contributions to the discussions. These notes, typed out as far as possible, were forwarded to the speakers with a request to check or to amplify them, in order to collect as much as possible of the remarks made. The discussions have been printed in part immediately after the

papers to which they referred, in part as separate chapters. In view of the variety of authors no attempt has been made to ensure complete uniformity in notation and in references.

The Symposium has led to an important deepening of insight into the problems of motion of the interstellar gas and into the physical phenomena which influence the behaviour of this gas. One result of great interest is the recognition of the importance of "compressible turbulence", and of the part played in it by shock waves. The statistical treatment of the random motion of such shock waves presents itself as a major theme for mathematical research. Another matter of great importance is the recognition of the decisive influence of gain and loss of energy through radiative processes; the picture of adiabatic motions apparently can be used in exceptional cases only.

Both Unions wish to express their gratitude to UNESCO for its help in the form of grants of money; the attitude of UNESCO with regard to meetings such as this one has been a great encouragement.

The hope has been expressed that a third Symposium may be organised on similar lines, after another period of 4 years, since new data concerning the problems discussed are continually forthcoming, while at the same time theoretical considerations are gaining in penetrative power and in clarity.

It has been agreed by the two Unions that the Proceedings of this Symposium should appear as Symposium Number 2 of the series "Reports on Symposia", published by the IAU.

The Editors wish to thank Mr. M. D. Frank, Director of the North Holland Publishing Co. at Amsterdam for the helpful way in which he has met their wishes and for the great care that has been taken to make the volume attractive.

Delft, J. M. BURGERS

23 August, 1954 Leiden, H. C. VAN DE HULST

CHAPTER 1

OPENING ADDRESS

BY

Sir GEOFFREY I. TAYLOR
Cambridge, England

I have been asked to open this Symposium, not, I imagine, because I know much about the high matters you will discuss, but because I am the fortunate person who was chosen by Batchelor, when he came to England, to guide his progress from practical problems in Aeronautics to the more recondite studies of Turbulence. It was from that restricted field that he first looked up and contemplated the grandeur of the Universe. This progress from the ridiculous to the sublime is one with which we in Cambridge are familiar; some people say that Eddington, one of our greatest alumni, pursued it in reverse when he reduced the whole thing to the number 137.

The progress of cosmological ideas seems to be analogous to that of the theory of turbulence. First came the phase in which individual particles and their reactions were considered. Then the effects produced by the concerted action of many particles were summarised in the kinetic theory of gases. When people came to study turbulent motion in fluids it was found that though the motion might, in one sense, be completely random like the motion of molecules in a gas, there was such an important element of concerted motion amongst the molecules that only the framework of the hydrodynamics of continuous fluids could be used to describe it. So in the cosmological problems which we will discuss there is a tendency to pass from particle dynamics to that of continuous fluids.

The fact that hydrodynamicists have solved a limited number of problems of continuous media gave rise to the idea in the mind of Burgers that they may have something to contribute to cosmological theory and so, being a man who never lets grass grow under his feet when he sees a chance of promoting useful scientific cooperation, he arranged the first symposium in Paris. Since that meeting, and no doubt largely because of it, there have been developments in theory.

There have also been great developments in our means of observing the cosmos. I need only mention the fact that the Schmidt telescopes and the 200 inch one have been very fruitful and that radio astronomy has been making great strides. About a year ago it seemed to Burgers and to some of his colleagues in Holland and America that the time was ripe for another symposium. Accordingly he got into touch with Batchelor and me to find out whether we could make local arrangements for a meeting in Cambridge in 1953. The present meeting is the result. I must confess that all the local work has been done by Batchelor, but I can speak for all of us in Cambridge in saying that we welcome most heartily the astronomers, physicists and hydrodynamicists who have come to us in many cases from great distances to join this symposium. I, for one, look forward to hearing of the many exciting new results which our programme leads us to expect.

PART I
THE OBSERVATIONAL DATA

CHAPTER 2

THE OBSERVATIONAL BACKGROUND OF COSMICAL GASDYNAMICS

BY

R. MINKOWSKI
Pasadena, Calif.

The general picture of the interstellar gas in the Galaxy today is essentially unchanged from that presented at the Symposium on the Motion of Gaseous Masses of Cosmical Dimensions [1] at Paris in 1949.

The Galaxy is a rotating stellar system containing stars, gas and dust. Unlike the bulk of the stars, gas and dust are highly concentrated towards the equatorial plane in a layer a few hundred parsec thick. In this respect gas and dust are similar to the bright stars of types O and B which are the most conspicuous representatives of the stellar population I. It is now generally recognized that these stars are short-lived compared to the age of the Galaxy. They must have been formed relatively recently, and probably are still being formed now, from the gas and dust by condensation; accretion also may play a role.

In the neighborhood of the sun, which is at a distance of about 8000 parsec from the center of the Galaxy in one of the spiral arms, the mass of the gas is of the same order as that of the stars, about 3.10^{-24} g/cm^3 or 2 H atoms/cm^3. That the gas is not smoothly distributed throughout the Galaxy has first become clear from the study of similar galaxies, such as the Andromeda nebula M 31. The gas, as well as the dust and the bright stars of types O and B, is concentrated in the spiral arms. Along any line of sight in the plane of the Galaxy, the large scale distribution of gas and dust is therefore not smooth; high density as in the solar neighborhood within the arms will alternate with very low density between the arms.

Very decisive progress in the study of the interstellar gas and its motions has been made since 1951 with the aid of the 21-cm radiation of neutral hydrogen, predicted by Van de Hulst in 1945 and discovered by Ewen and Purcell [2] and almost simultaneously by Muller and

Oort [3]. This radiation permits the observation of the previously unaccessible main component of the gas, neutral hydrogen, and thus furnishes the most powerful means to study the distribution and motions of the gas. With its aid many of the main features of the spiral structure have already been determined (see Ch. 4). Studies of the distances of O and B type stars by Morgan and of interstellar absorption lines by Münch support and supplement these results. But the resolving power achieved with the 21-cm radiation is still too low for a study of the structure and motions of the gas in its finest details. In this respect our knowledge still comes from the application of optical methods.

In its natural state—not excited to emission and ionised by the radiation from hot stars—the interstellar gas can be investigated optically only with the aid of the absorption lines of Na, Ca, Ca^+, K, Fe, Ti^+, CN, CH and CH^+ which appear superposed on stellar spectra. The information thus obtained is clearly extremely localised, referring only to the gas in a cylinder of stellar radius. Moreover, since only stars of early types can be used whose spectra show no interfering lines of their own, close spacing of observations in the sky is usually not possible. Most of the observations refer selectively to thin and diluted clouds. Stars in or behind dense and thick clouds are not easily accessible to spectroscopic observation with the required high dispersion since the dust contained in such clouds obscures the stars.

The study of the interstellar lines shows that the gas is predominantly neutral (H I region). The temperature is about 100° K in dense H I clouds. The average density is of the order 1 H atom/cm^3 in the spiral arms, much lower outside. The gas takes part in the rotation of the Galaxy, but individual clouds have random motions averaging about 5 to 10 km/sec. Occasionally higher velocities up to 100 km/sec occur. Velocities of approach seem to be more prevalent. The typical cloud has been pictured as having a density of 10 H atoms/cm^3 and a diameter of 10 parsec, with 5 to 10 clouds being present per 1000 parsec. Between the clouds a low density of the order 0.1 H atoms/cm^3 has to be assumed. This picture is certainly an extreme oversimplification.

Dust, probably formed by condensation from the gas, is intermixed with it. Obscuration by the dust, decreasing with increasing wave length, makes the clouds visible as dark areas which appear on the background of rich star fields or of bright nebulosities. The dust has probably little influence on the aerodynamical conditions. It makes

the structure and distribution of the interstellar clouds visible, but does not offer ways to determine their velocities. If, as seems possible, the ratio of gas to dust is not fixed, the density of the dust is not a quantitative indicator of the gas density. Starlight transmitted by the dust becomes polarized [4]. This effect is due to orientation of elongated particles. The most plausible explanation seems to be that the orientation is an effect of interstellar magnetic fields [5]. As an alternate interpretation, the effect of streaming of grains through gas has been proposed [6].

Dust clouds become visible as reflexion nebulae when they are illuminated by nearby bright stars. If the temperature of the stars is high enough, the gas becomes ionised (H II region) and becomes visible as an emission nebula. Roughly, stars of spectral type earlier than B 1 are the exciting sources of emission nebulae, but the division between emission and reflection nebulae is not sharp. It is not unusual to find faint emission lines in a nebula whose light is mostly due to scattering by dust. In general, reflection nebulae are relatively blue, since scattering increases with decreasing wavelength, while emission nebulae are relatively red, since H α is usually the strongest line.

In a typical emission nebula the gas is highly ionised, so that the density N_e of the electrons and N_H of hydrogen are practically equal. The surface brightness depends on the electron temperature T_e and is proportional to N_H^2 and to the thickness r of the emitting volume. Since to a first approximation T_e is always of the order 10000° K, the surface brightness is essentially determined by $N_H^2 \cdot r$, called emission measure by Strömgren [7]. The low value of T_e compared to the much higher temperature of the exciting stars is due to the cooling effect which results from the excitation of forbidden lines by electron collisions which reduce the mean kinetic energy. Typical bright emission nebulae represent dense clouds excited by one or several stars in or very close to the cloud. An exciting star within a cloud can ionise the gas only out to that distance at which the ionising radiation is completely absorbed. Thus an H II region obtains a radius s which depends on temperature and luminosity of the star and on the density of the gas. A few typical values of the radius s of the ionised region, the Strömgren sphere, are (s in parsec; N_H in cm^{-3}):

Spectral type of exciting star	O 5	O 7	B 0
s	$140\ N_H^{-2/3}$	$86\ N_H^{-2/3}$	$26\ N_H^{-2/3}$

A bright emission region shows the complete gas cloud only if the cloud

is smaller than the Strömgren sphere. In individual cases it is not always easy to decide whether the Strömgren sphere or a whole cloud is seen. In addition to the typical emission nebulae, the existence of large extended areas of faint emission has been shown first by Struve and his colleagues. The excitation of such extended H II regions is mostly due to the radiation field of many stars rather than to a single star. From the observed emission measures values of N_H are found which range from 10^4 to 10^5 cm^{-3} for the central part of the Orion nebula, an exceptionally dense cloud, to 1 cm^{-3} for the faintest extended emission regions.

The study of the appearance of obscuring clouds and emitting gas has benefitted greatly by the use of Schmidt telescopes which offer the possibility to observe large areas of the sky with good definition in different colors. Particularly useful are exposures with relatively restricted spectral range in the red. Since H α is usually the strongest line and the interstellar absorption decreases towards the red, very faint emission nebulosity becomes observable which is invisible on plates taken in the blue. Absorbing clouds actually have less contrast in the red than in the blue. But relative to the sky, whose brightness limits the exposure time, the background of stars and emission nebulosity is brighter in the red where the absorption is lower. For this reason dark clouds can be observed better in the red.

If one looks at the central Milky Way*, it becomes at once obvious that the schematic picture of separate clouds of 10 parsec diameter has little resemblance to reality. An entirely chaotic mass of dark clouds of all possible shapes and sizes appears projected on the background of stars and faint emission nebulosity. The linear scale of the details cannot be assessed easily. Some idea of the scale can be obtained from the bright emission nebulae in the area. The Trifid nebula (NGC 6514)

* Prints and slides, mostly from plates obtained with the 48 inch Schmidt telescope on Palomar Mountain for the National Geographic Society — Palomar Observatory Sky Survey, were shown at the symposium. Since it is technically impossible to include them in this report, reference is made to: E. E. Barnard, A Photographic Atlas of Selected Areas of the Milky Way, edited by E. B. Frost and M. C. Calvert, Carnegie Institution of Washington, 1927. In particular, reference is being made to absorption detail by using Barnard's catalogue of "Dark Objects in the Sky" which lists these objects by a number preceded by the letter "B". It should be noted that Barnard's atlas shows photographs in the blue with smaller scale and poorer definition than that now obtainable with Schmidt telescopes. This atlas is used here only as an easily accessible source of identification which gives not always an adequate picture of detail under discussion.

is a typical emission nebula at a distance of 700 parsec. At this distance the angular diameter of about 30′ corresponds to a linear diameter of about 6 parsec, somewhat smaller than that of the "typical" cloud. If the nebula is the Strömgren sphere of the exciting star of type O 7, which seems probable, the density of the cloud is roughly 160 cm^{-3}. Although some dark clouds can be seen which may have the approximate size of the "typical" cloud which is indicated by the Trifid nebula, they are certainly not a prevailing feature. Instead, huge complexes of clouds covering many square degrees are seen, broken up in numerous irregular details. Very noticeable are also some small clouds with very heavy obscuration, often with very sharp boundaries.

These small clouds are in some respects very similar to the absorption detail which appears projected on bright emission nebulae. The dark cloud B 92, for instance, shows a very dense cloud with a very sharp eastern edge. It cannot be at a very large distance since it is virtually empty of stars. It may well be quite similar in size to apparently much smaller features of this type in emission nebulae which it simulates in shape. Only occasionally, however, can very small condensed clouds be found which are comparable to the finest detail superposed on some emission nebulae in large numbers. Equally rare are such remarkable objects as B 75 and B 84 which show tentacles very similar to the spikes or "elephant trunks" which are not unfrequent on the background of bright emission nebulae. It is not certain whether the features seen projected on the Milky Way are larger than their counterparts in emission nebulae. It is possible that very small clouds escape detection on the stellar background, but it does not seem likely that this explains their scarcity. It should be noted that large areas of the central Milky Way show a veil of very faint emission nebulosity. This may be important if certain features of dense clouds depend on their being embedded in ionised gas of high temperature as their frequent occurrence in H II regions suggests.

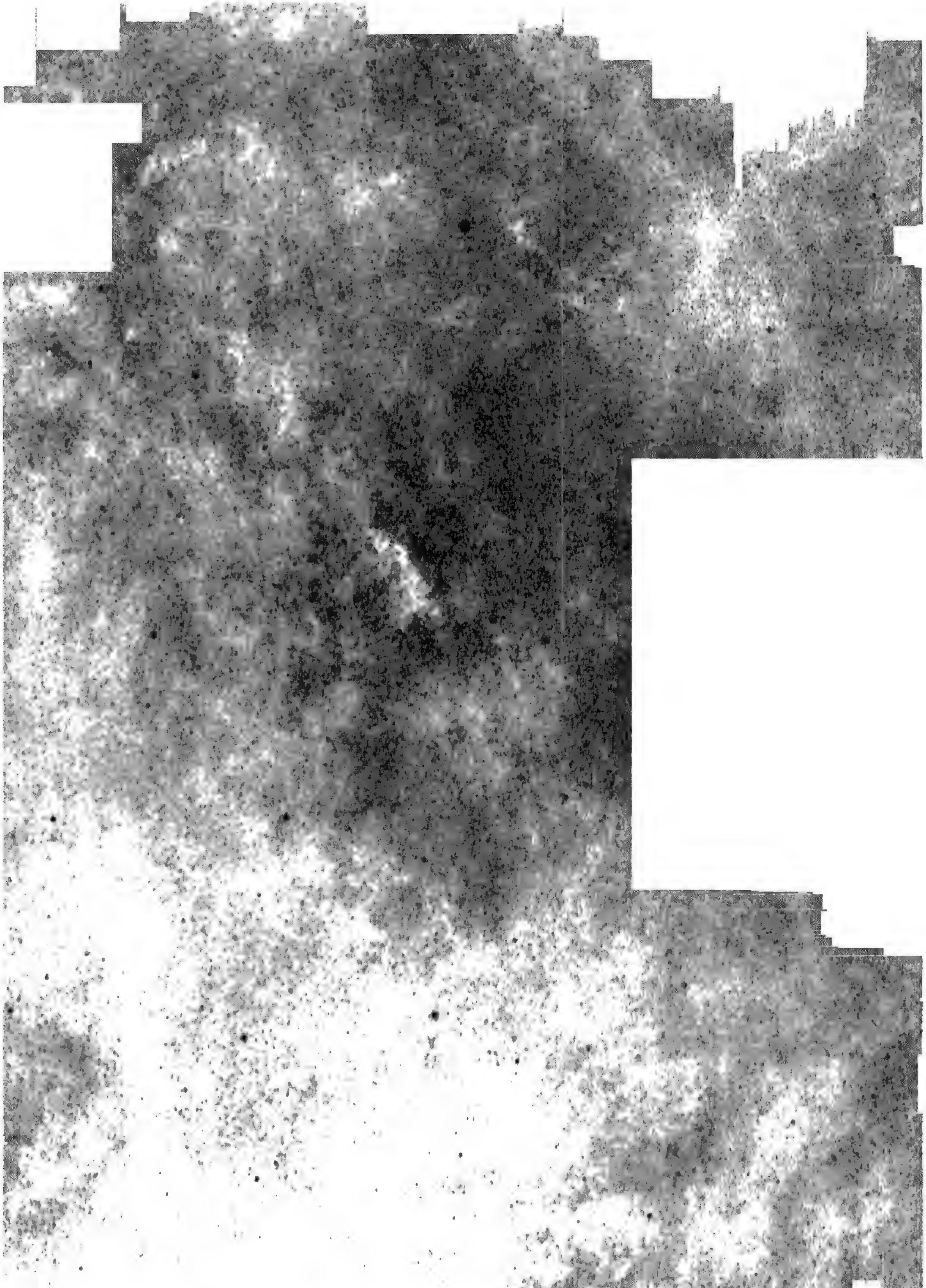
More characteristic for the central Milky Way than small and sharp details are diffuse and often elongated clouds, such as the famous lane east of ρ Ophiuchi (Barnard, plate 14) which extends with a width of about 40′ for more than 6° and continues for several degrees in broken clouds. Curved and twisted clouds are not unfrequent. Good samples are the *S*-shaped marking B 72 or the twisting dark lane 2° north of 58 Ophiuchi which seems to end in the west with the tentacled cloud B 84. Systems of nearly straight lanes frequently give the impression of a flow pattern. Sometimes such patterns running in different

directions are superposed. Such systems may also consist of many small diffuse clouds, as in the area, shown in Fig. 1, which almost simulate a cirrus cloud. The patterns do not follow a fixed direction but are obviously curved.

Reflection nebulae show usually only rather diffuse structures with little sharp detail, as is to be expected since they are merely illuminated clouds of basically the same character as the absorption clouds. The outstanding example of a reflection nebula with extremely fine detail is the Pleiades nebulosity with its sharp striations. Individual streaks are about 0.005 parsec wide, about 0.5 parsec long. The nebulosity extends faintly to a considerable distance, particularly to the east. The directional quality of the nebulosity remains clearly visible, but the fine structure is absent. This may raise the question whether the bright stars of the cluster influence the structure of the cloud in which they are embedded.

Large emission regions show many details which are of very similar character as the absorbing clouds. The region of the North America nebula and the nebulosities near γ Cygni are one of the best examples. A small part near the western rim of this area is shown in Fig. 2. All the nebulosities in the region seem to be part of one huge mass, cut in two main parts by heavily obscuring clouds along the galactic equator. The emission is excited by many O and B type stars at a distance of 700 parsec. If this is also the distance of the nebulosities, they fill a diameter of 250 parsec. In the brighter condensations N_H is probably of the order 30 to 50 cm^{-3}. The average density of the whole mass is obviously much lower, probably not much larger than 1 cm^{-3}. Towards its outer edge, particularly on the western side, the character of the nebulosity changes. The chaotic structure is replaced by filamentary structures and striations which are many degrees long. The filaments become exceedingly sharp north of the area shown in Fig. 2. These structures seem to envelop the area rather than follow a fixed direction; in the upper part of Fig. 2 the direction is approximately parallel to the galactic equator. It is tempting to assume that the formation of these structures near the edge of the mass is the result of an expansion of the mass of gas into space of much lower density.

Fig. 1. Region in Ophiuchus; center 17^h44^m, $-24°0'$. Photographed with 48 inch Schmidt telescope, λ 6200 to 6600, scale 1 mm = 66″ (1.01 × original size). Copyright National Geographic Society—Palomar Observatory Sky Survey.



The brighter condensations of the whole mass are very rich in fine absorption detail of the kind typical for bright emission nebulae: sharply bounded clouds, many with "elephant trunks", and very small, opaque and therefore probably very dense clouds. Particularly rich in such details is the area west and south of the North America nebula, which is shown in Fig. 8. It contains one of the outstanding "elephant trunks", in an area too much overexposed to show it in Fig. 8. Remarkable striated absorption features are in this area. Two systems are superposed which form nearly a right angle. The finer striations of the north-south system begin to approach in sharpness the striations in the Pleiades nebula, but on the whole the structure is more diffuse.

Typical bright emission nebulae show in many cases not the whole cloud, but only the Strömgren sphere of the exciting star or stars in the center. The bright central region is often surrounded by extended, sometimes extremely faint emission nebulosity. This can indicate that the bright central part is really a central condensation of high density, but it also may be the result of weak excitation of parts outside of the Strömgren sphere by more distant stars. If a bright nebula is in the center of a dark cloud, as for instance I C 5146, the interpretation of the luminous part as a Strömgren sphere seems obvious.

Only rarely are all data available which are necessary to discuss this question. The nebula NGC 2237–39, surrounding the open cluster NGC 2244 and excited by its O and B type stars, may serve as an example [8]. The nebula appears as a broad ring with an average surface brightness of $1.3 \cdot 10^{-2}$ erg cm^{-2} sec^{-1}. At a distance of 1400 parsec the linear diameter is 35 parsec. On the assumption that T_e equals 10000°, the surface brightness corresponds to $N_H = 13.5$ cm^{-3}. On the other hand, if the border of the luminous area is the edge of the Strömgren sphere, N_H would be 15 cm^{-3}. The agreement is good; it would become complete if the electron temperature were slightly higher. If in this case the bright part would not be the Strömgren sphere, it would have to be assumed that by mere accident the dense part of the cloud has the exact size of it. Spectroscopic investigation shows that the low intensity in the center is not due to very high ionisation of the central region, but indicates low density. The nebula thus seems to be of the same general type as ringshaped nebulae in other galaxies. Its diameter is of the correct order of magnitude. A natural explanation of the ring shape would be expansion of the hot gas in the H II region (cf. Ch. 28).

The nebula also shows a rich collection of small and dense dark

clouds. The smallest of these have diameters of less than 5′, corresponding to 0.085 parsec. Very often sharply bounded dark clouds seen projected on emission nebulae have luminous edges, similar to the Horse Head nebula, the most familiar example of this phenomenon. In some nebulae, such luminous edges are very frequent. The nebula IC 1805, a very inconspicuous object in the blue, appears in the red as an area surrounded by bright edges, all of them pointing inwards. As a rule, typical bright edges seem to appear only on that side of a cloud which points toward the exciting star. There is at present no evidence that the excitation from the star does not provide an adequate explanation of luminous edges. The exciting radiation cannot penetrate deeply into a dense cloud. If only the outermost layers of a cloud are excited, a luminous edge may be seen, just as in a cumulus cloud properly illuminated by the sun. Thus, while the shapes of the dark clouds may be and probably are determined by hydrodynamical forces, the excitation may be provided by a star. If, on the other hand the excitation is due to mechanical heating of the boundary by the interaction between cloud and surrounding gas, expansion of the H II region would explain why luminous edges point inwards. Unfortunately, a clearcut decision between radiative and collisional excitation by spectroscopic observations is usually not possible (see the discussion in Ch. 18).

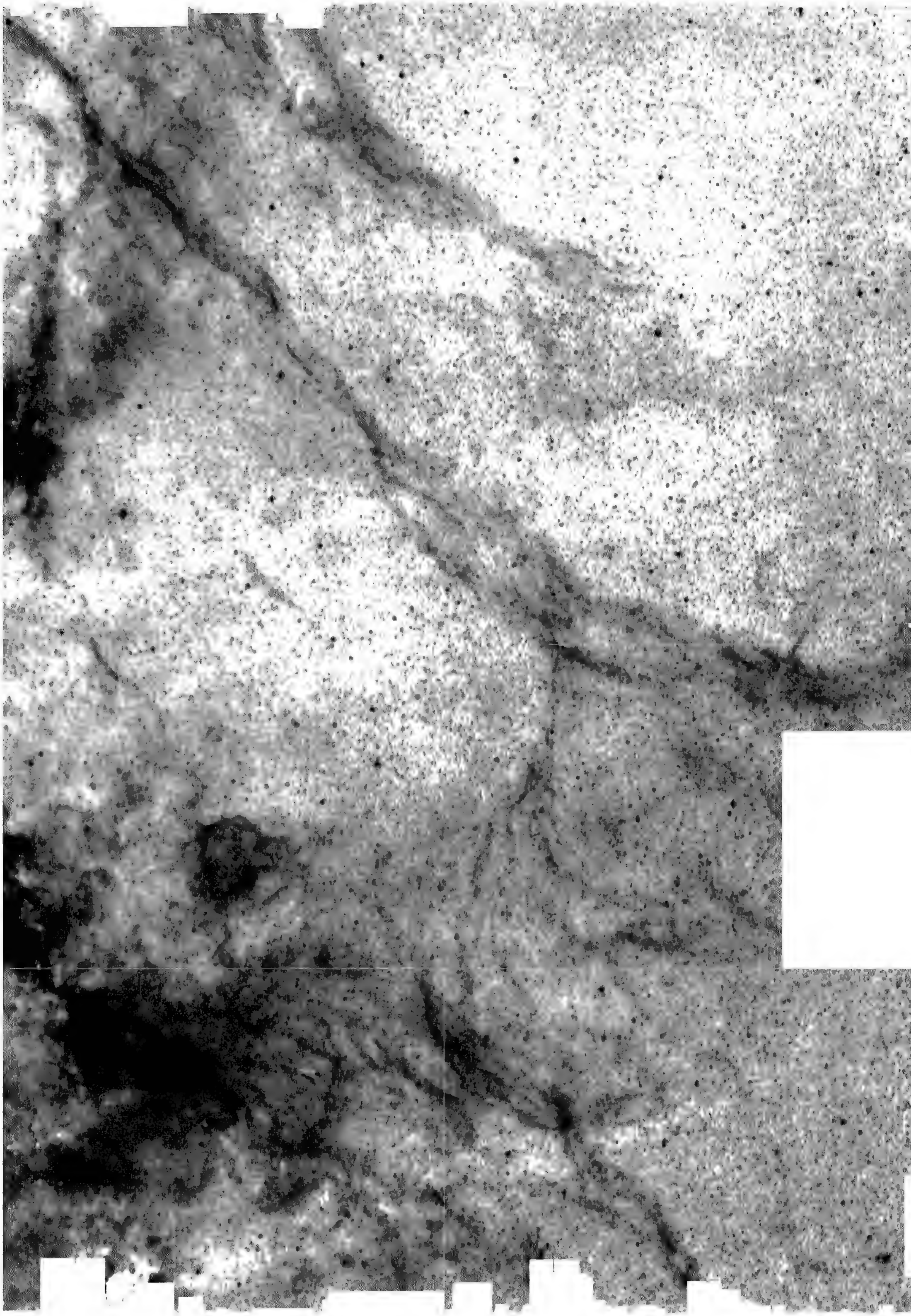
Some information on the interstellar gas may be obtained from objects which are not part of it. Interaction with the interstellar gas has been suggested for certain objects which are masses of expanding gas. Little is to be added to the discussion of these objects by Oort [9].

The origin of the great loop in Cygnus is still not definitely established. The remarkable filamentary structure of the nebulosity, which may have a counterpart in IC 443, has not yet found a full explanation. Interaction with the interstellar gas may be the main factor. However, the radiation of the gas does not seem to be excited by heating due to mechanical causes. A discussion of the spectrum (Ch. 18) leads to the conclusion that the excitation is provided by radiation from a hot star.

To Oort's discussion of the shell of Nova Persei (1901) nothing can be added at this time.

The only supernova shell which can be investigated in detail is the Crab nebula. It consists of two parts, an inner diffuse mass which shows

Fig. 2. Region in Cygnus; center 20^h53^m, $+42°15'$. Photographed with 48 inch Schmidt telescope, λ 6300 to 6600, scale 1 mm = 94″ (0.71 × original size).



a continuous spectrum without observable emission lines and a surrounding mass of filaments which show emission lines resembling the spectra of ordinary emission nebulae, but the H lines in this spectrum seem to be relatively weak compared to the lines of He an He^+ as well as to the forbidden lines of heavier ions. This may be an indication that the hydrogen abundance is low. The theory of collisional excitation has not been developed to a stage which would permit a discussion whether collisional excitation of the filaments might play a role. (New spectroscopic evidence obtained after the symposium shows that the irregular shape of the outer edge of the filaments is caused by random motions up to several hundred km/sec superposed on the expansion with a velocity of 1100 km/sec. No evidence for retardation at the edge can be found.) A new discussion of the observations [10] shows that the electron temperature in the central mass may be as high as 2.10^6 degrees. At such a temperature conversion of internal mechanical energy might be an important factor for the excitation of the inner mass. Its state of motion cannot be observed in the absence of emission lines. But high internal motions are suggested not only by the diffuseness of the inner mass, but also by the strong non-thermal radio emission of the Crab nebula since such emission seems to be connected with the occurrence of large random motions.

Another class of objects which might show effects of interaction with the interstellar gas are planetary nebulae. These expand with velocities of the order of 30 km/sec. The densities range downward from 10^4 cm^{-3}. As Zanstra has first remarked, the border of the gas is not visible in bright planetaries which are optically thick for the exciting radiation, which is exhausted before the edge is reached. Since with progressing expansion the gas density and the optical thickness decrease, exciting radiation will finally reach the border and begin to leak out. This might explain why some planetaries, such as the ring nebula in Lyra (NGC 6720), show extremely faint envelopes surrounding a bright central region. In a still later phase, the exciting radiation will penetrate the gas completely with an ever increasing loss of exciting energy. The nebula will then have very low surface brightness, but the entire gas will be seen. It seems probable that nebulae in this phase are among planetaries of extremely low surface brightness which are being found with the 48 inch Schmidt telescope. The faint envelopes of bright planetaries and many low surface brightness planetaries show outer edges which are brighter than the adjacent inner parts. It is tempting to assume that such bright edges are the

result of compression and possibly excitation by the interstellar gas into which the nebulae expand.

Finally, non-thermal radio sources should be mentioned. The strongest such source in Cassiopeia is a gaseous nebula of a highly peculiar and hitherto unknown type. Its outstanding trait are the extremely high random motions of the diffuse small clouds. Their radial velocities range from −1800 to +5000 km/sec, completely hiding any expansion of the mass. A velocity dispersion of several hundred km/sec is found in individual clouds of this type. Co-existing with the diffuse clouds are sharp and smaller bits of gas, with internal velocities of smaller spread. Their radial velocities range between 0 and 100 km/sec. It is possible that they form a slowly expanding system. The second strongest source, Cygnus A is of entirely different type. It is a pair of galaxies in actual collision. In such a collision strong interaction must occur between the gas masses of the systems. No convincing explanation of the intense radio-emission has been proposed, but it is obvious that interaction of fastmoving gas clouds, probably under the influence of magnetic fields, must be the source of non-thermal radio emission.

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Fig. 8. Region in Cygnus; center 20^h55^m, $+42°30'$. Photographed with 48 inch Schmidt telescope, λ 6200 to 6600, scale 1 mm = 66" (1.01 × original size). Copyright National Geographic Society Palomar Observatory Sky Survey.

CHAPTER 3

EXTRAGALACTIC STELLAR SYSTEMS

BY

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Stellar systems outside our own system (the galaxy) are important for the study of the behavior of gas and dust, as they provide information on its large scale distribution. There appears to be a large variety of structural patterns among the extragalactic systems. It seems very likely that they have a bearing on their dynamics and on the stage of evolution.

At the symposium, an introduction on this subject was given by the late Dr. E. P. Hubble, who died shortly after the meeting. At the request of the organizing committee, the present author undertook to write a short account of the main points of interest for cosmical aerodynamicists. Not without hesitation, however, as it seems hardly justified that anybody not experienced in the observation of extragalactic nebulae should try to present such an account. The justification for the present report lies mainly in the fact that none of the other participants seemed especially entitled to the task, that Dr. Hubble limited himself mainly to a description of his system of classification which has been fully described in his monograph, "The Realm of the Nebula", and that the present writer had the privilege to inspect a part of Dr. Hubble's large collection of photographs of the brighter nebulae a few years ago during a stay at Pasadena.

THE HUBBLE MEMORIAL VOLUME

Dr. Hubble realized that only a few workers, both theoretical and observational, have had the opportunity to study the detailed structure of nebulae as shown by large scale photographs. He therefore formulated a plan which would make available to astronomers a photographic catalogue of extragalactic nebulae. After Dr. Hubble's death Dr. Allan

* Now at Yerkes Observatory, Williams Bay, Wis., U.S.A.

Sandage of the Mount Wilson and Palomar Observatories, has undertaken to assemble the data and to write the text for this atlas. The atlas will be published as a memorial volume to illustrate as fully as possible the details of Hubble's system of classification. The following information was kindly supplied by Dr. Sandage to the author of this article:

"...Hubble's sequence of nebular classification, which is universally adopted, has never been adequately illustrated. Indeed, as more and more plates were accumulated after his initial description of the classification, Hubble changed the details of the scheme for the transitional nebulae S0, Sa and SBa. This change came about from inspection of the subtle, incipient, structure of those forms which appeared only on the best 100-inch and especially 200-inch plates. These structural details have never been published.

"The catalogue should be thoroughly representative. It should (1) illustrate all basic forms *along* the sequence of classification and (2) should show, within each specific type, the variations *across* the sequence. Besides the photographs, some text is desirable which would contain a rather detailed account of Hubble's classification system so that the descriptions may be directly correlated with photographs of the objects defined as prototypes.

"We estimate that there will be 150 or 200 individual nebulae which should be illustrated. Perhaps half of these can conveniently be reproduced so as to give four illustrations per printed page. One quarter of the total can be of a size so as to give two prints per page. The remaining quarter will require full page illustrations. This estimate gives 100 pages of the catalogue for reproductions. An additional 20 to 30 pages of descriptive text and discussion would complete the volume

"Plates of all the objects, taken with the 100-inch telescope, are now available (April 1954). A number of 200-inch plates are also available. In a few instances it will be desirable to obtain 200-inch plates on critical cases where only plates with the 100-inch now exist. The number of such cases is, however, small."

In the following we shall in the first place give a summary of Hubble's classification of extragalactic nebulae, next describe some features of the spiral nebulae that seemed particularly striking during the inspection of Dr. Hubble's plates, and add some references to published investigations which seem of interest for the student of interstellar matter in stellar systems.

CLASSIFICATION OF EXTRAGALACTIC NEBULAE

The following describes the classifications into the types introduced by Hubble. The quotations are from Chap. II of his book, "The Realm of the Nebulae" [1], to which we also refer for illustrations of the sequence of classifications. "As a first step the nebulae are divided into two very unequal groups. The great majority are called 'regular nebulae', since they exhibit, as a common pattern, conspicuous evidence of rotational symmetry about dominating, central nuclei. The remaining objects, about 2 or 3% of the total number are called 'irregular', because they lack both rotational symmetry and, in general, dominating nuclei.

"Regular nebulae are either 'elliptical nebulae' or 'spirals'. Objects in each group fall naturally into ordered sequences of structural forms; and one end of the elliptical sequence is rather similar to one end of the spiral sequence. Accordingly, the two sequences are oriented, for purposes of description, as though they were two sections of a single larger sequence containing all structural forms encountered among the regular nebulae. The zero point is arbitrarily selected at the free end of the elliptical section. The progression throughout the complete sequence thus runs from the most compact of the elliptical nebulae to the most open of the spirals, a progression in dispersion or expansion. The terms 'early' and 'late' are used to denote relative position in the empirical sequence without regard to their temporal implications. These emphasize the purely empirical nature of the sequence of classification..."

It may be stressed that Hubble's last remark still holds: There is no clear evidence yet as to which type of spiral nebula represents the young or the late stage in the development of spiral structure or whether Hubble's spiral sequence represents a sequence of ages at all.

ELLIPTICAL NEBULAE

"Elliptical nebulae are designated by the symbol E. They range from globular objects through ellipsoidal figures to a limiting lenticular form with a ratio of axes about 3 to 1. It is probable that all regular nebulae with main bodies flatter than this limiting form are spirals..."

SPIRALS

"Spiral nebulae fall into two distinct but parallel sequences, containing normal and barred spirals, designated as S and SB, respectively.

A thin scattering of mixed forms lies between the series. In the normal spiral, the two arms emerge smoothly from opposite segments of the periphery of a nuclear region resembling a lenticular nebula, and thence wind outward along spiral paths. In the barred spiral, the two arms spring abruptly from either end of a bar of nebulosity stretching diametrically across the nuclear region, and thence follow spiral paths similar to those found in the normal spirals. Normal spirals are more frequent than barred forms in the ratio of 2 or 3 to 1."

Normal Spirals

"At the beginning of the sequence, the normal spiral exhibits a bright, semistellar nucleus and a relatively large nuclear region... which closely resembles a lenticular (E7) nebula. The arms which emerge from the periphery are... closely coiled. As the sequence progresses, the arms increase in bulk at the expense of the nuclear region, unwinding as they grow until in the end they are widely open and the nucleus is inconspicuous. About the middle of the sequence, or slightly earlier, condensations begin to form. The resolution generally appears first in the outer arms and gradually spreads inward until, at the end of the sequence, it reaches the nucleus." (See NGC 2841 and NGC 628, Fig. 1 and 2).

Barred Spirals

"The barred spiral is first seen as a lenticular nebula in which the outer regions have condensed into a more or less conspicuous ring... concentric with the nucleus, and a broad bar has condensed diametrically across the nucleus from rim to rim. The appearance resembles that of the Greek letter theta, θ. As the sequence progresses, the ring appears to break away from the bar at the two opposite points, just above the bar at one end and just below the bar at the other (compare NGC 1300, Fig. 8) and the spiral arms grow out of the free ends of the broken ring. Thereafter the development parallels that of the normal spiral; the arms build up at the expense of the nuclear region, unwinding as they grow; resolution appears first in the outer arms and works inward toward the nucleus. The last stage is the familiar *S*-shaped spiral with thin, well-resolved arms (NGC 7479)."

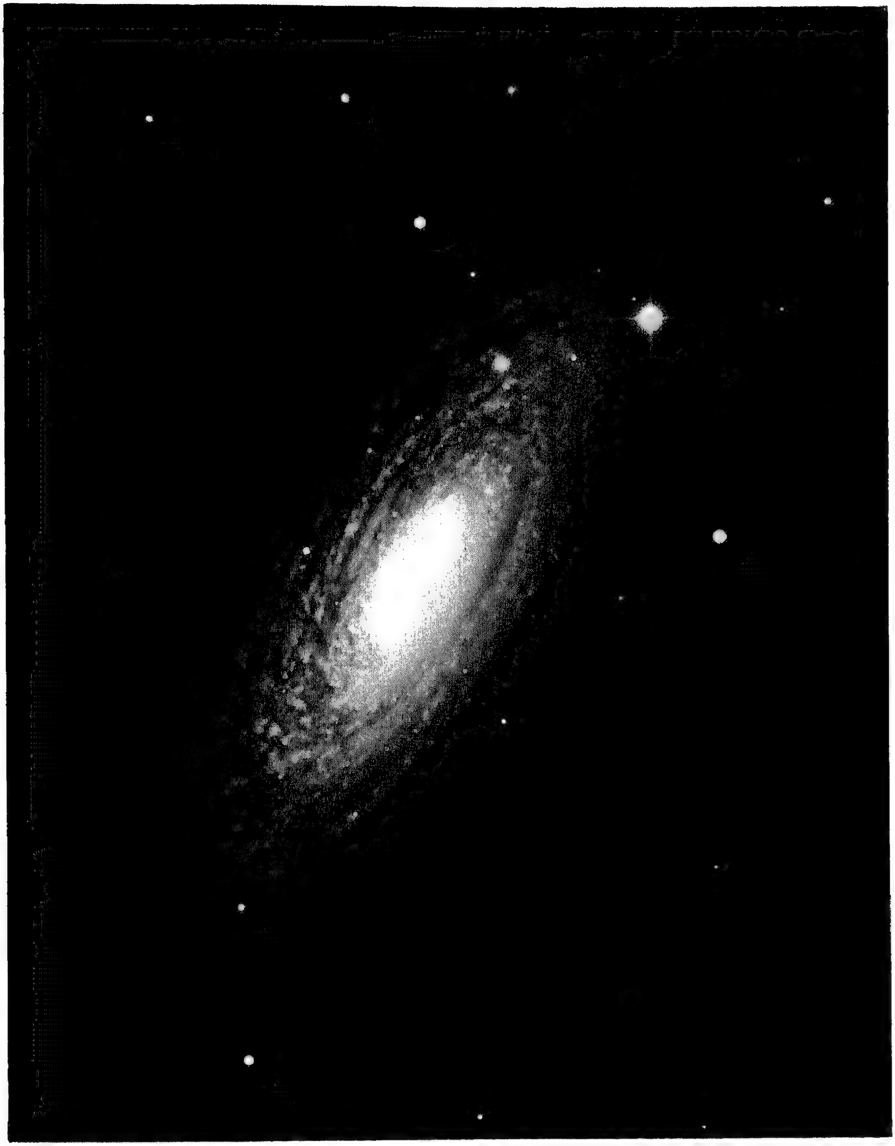

Fig. 1. NGC 2841, Hubble's type Sb. Note narrowly wound spiral arms around extended nucleus. 200″ Photograph, scale 1 mm = 3.″6.

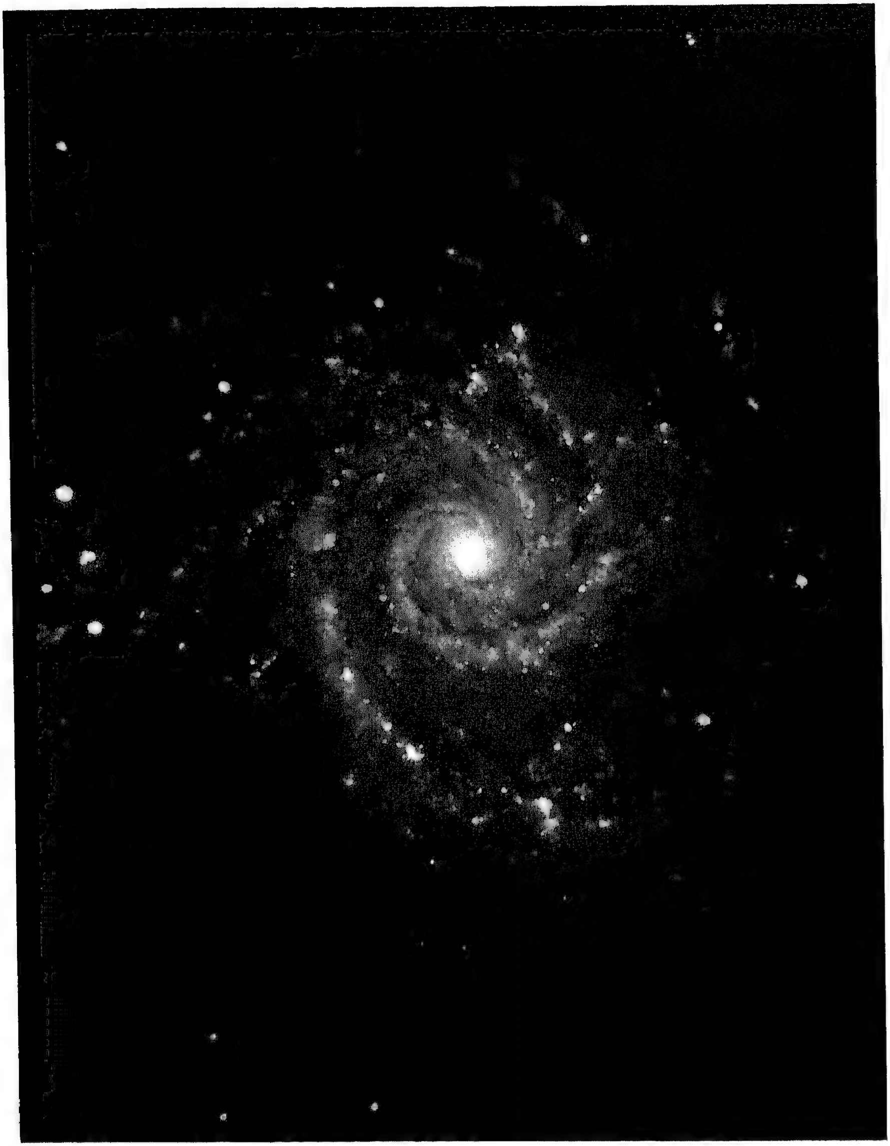

Fig. 2. NGC 628, Hubble's type Sc. Note dust lanes and patchiness of spiral arms. 200″ Photograph, scale 1 mm = 4.″5.

Sequence of Spirals

"Progression in the two series is fairly indicated by the relative luminosity of nuclear region and spiral arms, by openness of the arms and by the degree of resolution...

"Provisionally, each sequence of spirals has been subdivided into three sections designated by the subscripts a, b, and c. Thus Sa, Sb, and Sc represent, early, intermediate, and late types of normal spirals and SBa, SBb, and SBc, represent the corresponding types of barred spirals... and nebulae intermediate between E7 and Sa are occasionally designated as S0..."

IRREGULAR NEBULAE

"Other nebulae, between 2 and 3% of the total number, show no evidence of rotational symmetry and hence do not find a place in the sequence of classification. These objects are called irregular nebulae and are designated as Ir. About half of the irregulars form a homogeneous group, in which the Magellanic Clouds are typical examples, and their importance probably merits a separate division. Since their stellar contents resemble those of very late-type spirals, they are sometimes considered as representing the last stage in the sequence of regular nebulae. Their status, however, is speculative and the absence of conspicuous nuclei may be of more fundamental significance than the absence of rotational symmetry, which is a possible consequence..."

THE DISTRIBUTION OF GAS AND DUST

In their article in the report on the first symposium on cosmical aerodynamics, Baade and Mayall [2] have pointed out that gas and dust are recognized under very different circumstances and, therefore, must be distinguished in the discussion of their distribution. Dust reveals itself by absorption of background light; gas by emission spectra.

From observations of edge-on spirals it appears that the dust usually is distributed in a thin layer highly concentrated to the equatorial plane of the system. This is similar to the strong concentration of interstellar matter towards the plane in our galaxy.

The distribution of gas, as revealed by the emission nebulae, can be followed only in so far as highly luminous stars are present which excite the gas. It is generally assumed now that these highly luminous stars are

formed in the gaseous medium and that the distribution of the bright stars, therefore, reflects the general distribution of the gas. As these bright stars define the spiral pattern in the nebulae, it is inferred that the gas is also distributed in spiral arms. In the case of our galaxy this is demonstrated very clearly by the radio measurements of the 21-cm hydrogen emission.

As was described by Baade and Mayall, the interstellar medium in the central part of the Andromeda nebula is shown especially by the absorption pattern. Dark lines emerge from the central region and gradually change into the chain of blue stars defining the further course of the spiral arm. Evidence from the Andromeda nebula and our galaxy as well as theoretical considerations suggest strongly that gas and dust, on the whole, are mixed so that they define the same spiral pattern. One might, therefore, assume as a working hypothesis in studying the extragalactic nebulae, that the distribution of the dust also indicates that of the gas. This renders particular interest to those spirals where the distribution of dark matter can be followed into the nuclear regions of the system, and to those where the shape of the dust arms seems somewhat different from that of the main arms. A discussion of dark clouds in galaxies was already given by H. D. Curtis in 1918, and by Lindblad [3] at the first symposium, with special reference to NGC 1300 (M 63) and NGC 3718, and in various publications of the Stockholm Observatory.

The transition of the bright outer arms into the dark arms in the central regions is also very apparent in other cases, like NGC 4303 (M 61), NGC 4321, NGC 1365, NGC 3184.

STRUCTURAL FEATURES IN GENERAL

Particularly striking from an inspection of the Mount Wilson collection of photographs of nebulae is the large variety of shapes of spiral nebulae. One cannot escape the impression that the emphasis in hitherto published pictures of spiral nebulae is so much on the cases of regular and pronounced spiral arms in late type spirals like M 31, M 51, M 81, M 101, that the sample they represent on the whole is a special one and somewhat misleading.

Randers [4] has already drawn attention to the tendency, found among many types of nebulae, to show circular rings or a system of rings. Sometimes these rings are the only noticeable feature of the nebula next to the bright nucleus (NGC 3081).

In other cases the ring seems to separate the inner part of the nebula,

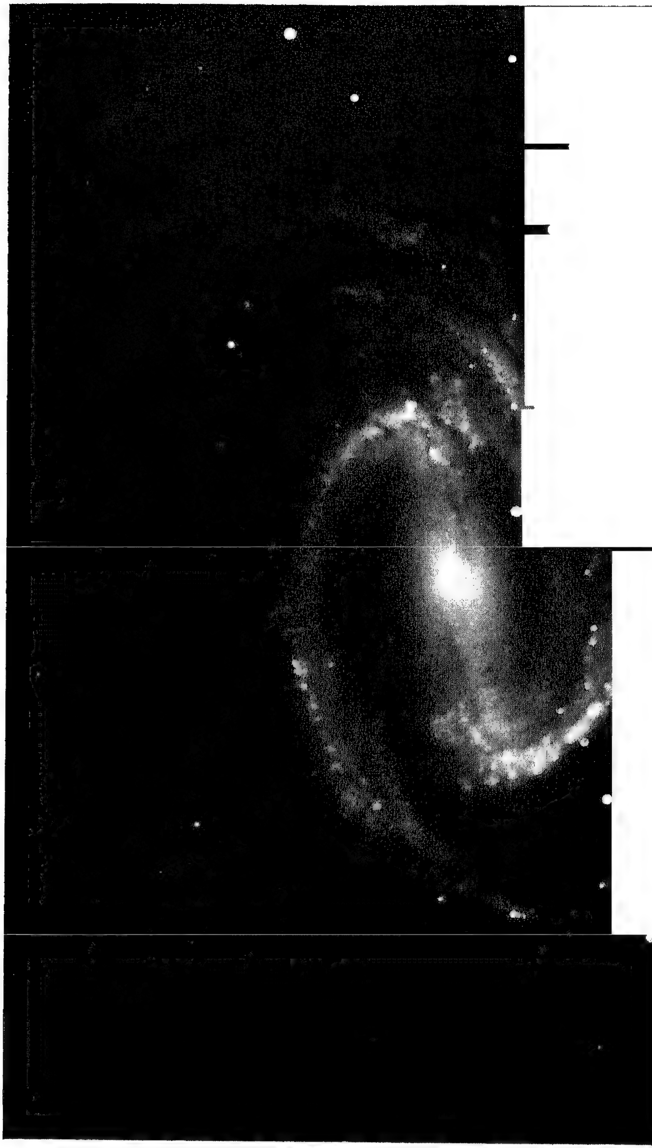

Fig. 3. NGC 1300, Hubble's type SBc. Note small central condensation anc lanes. 200″ Photograph, scale 1 mm = 4.″5, approximately.

with or without its own inner spiral structure, from the outer spiral structure. The arms in the outer part—usually two—wind from the ring and *not* from the nucleus as does the dark matter in the cases referred to above (NGC 1357, 2545, 2776, 2889).

The inner structure within the ring, when present, may consist of a bar (NGC 521, 638, 1358) or of a sometimes very pronounced and regular spiral structure. A striking case is NGC 1530, where this inner spiral structure seems especially pronounced in two dark arms emerging from the nucleus.

Other cases with markedly separate inner and outer spiral structure are NGC 4314, where two faint outer arms trail from a bar within which there is a marked spiral structure defined by the dark as well as by the brightest matter, but somewhat rotated with respect to each other; NGC 3277 with dark spiral structure in the inner part, whereas two outer spiral arms emerge from the edge of this; and NGC 3301.

Among the late type spirals, some show a large multitude of arms or rather portions thereof, and not just a few arms which can be followed from the centre. Sometimes this is also observed in the case of a ring breaking up into many slightly inclined fragments (NGC 278). Many arms emerging from the central ring are found in NGC 2532; other cases (NGC 1425, 3147) show many faint secondary spiral shaped connections between the main arms, or just many thin arms running from the edge of a central disk (NGC 3486). A somewhat different class seem to form the nebulae where two main arms show many branches, almost equally heavy as the main arms (NGC 2276).

The above notes do not give anything like a complete impression of all types. They are rather intended to stress the enormous variety of patterns of spiral nebulae which it will be very important to study in more detail to both astronomers and aerodynamicists interested in the subject.

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CHAPTER 4

INFORMATION ON VELOCITY AND DENSITY DISTRIBUTION IN THE INTERSTELLAR GAS DERIVED FROM ABSORPTION LINES AND 21-cm RADIATION

BY

J. H. OORT

Leiden

As has been impressively shown by Minkowski the distribution of interstellar matter is extremely complex. The observation of dark clouds in particular indicates that the distribution can be more adequately described by a picture of individual clouds separated by large spaces of negligible density, than by that of a more or less continuous medium with turbulent-like motions. The observation of radial velocities points to the same fact. The interstellar absorption lines observed in distant stars are not simply broadened by a continuous "spectrum" of internal motions, but they can be resolved into a number of discrete lines which are most easily interpreted as due to movements of individual clouds.

Attempts have been made to use the data on dark clouds and on multiple interstellar lines for rough statistics concerning sizes, densities, frequency in space and motions of clouds. A picture that is often used is that of clouds with a mean diameter of the order of 10 parsec and a density of 10 hydrogen atoms per cm^3. In the spiral arms there would be about 1.8×10^{-4} of such clouds per $parsec^3$, a line of sight of 1 kiloparsec cutting about 10 clouds. The clouds would fill about 7 % of space. The reality is far more complex, however. Individual clouds differ enormously in size. They appear, moreover, to have some tendency to form large agglomerations with masses of several hundred "normal" clouds.

A datum that is of interest for this symposium is the velocity distribution. Information on this has been obtained from the curve of growth of interstellar lines in different longitudes, and later from the multiple interstellar lines. I wish to refer to investigations by Blaauw [1] and by Schlüter, Schmidt and Stumpff [2].

Blaauw finds that the distribution of the random cloud motions can be represented by the expression

$$\frac{1}{2\eta}\, e^{-|v|/\eta},$$

where v is the radial velocity and η is the average random motion in one co-ordinate. For η he found values from 5 to 8 km/sec.

The Göttingen authors discuss the possibility that the lines of higher velocity arise in shells expelled by the stars in which they are observed. It appears more probable to me that most of these are not circumstellar, but are absorbed in the large cloud complexes in which expanding motions have been set up by the formation of an ionized region within these agglomerations (cf. Ch. 28).

Important information has recently been derived from observations of the emission line of hydrogen at 21-cm wave length [8]. This line is emitted by the neutral hydrogen atoms that populate the cool clouds. About 90% of the interstellar hydrogen is in this neutral atomic state. When we observe the 21-cm line in the direction of the centre of the Galactic System, or in the opposite direction, we find it considerably widened by random motions. When the "optical depth" is known the velocity distribution can be deduced from these line profiles. Similar deductions can be made at other longitudes, in particular between 65° and 180°. Here the line is greatly broadened in the direction of the high frequencies, due to the effect of differential galactic rotation. In addition we find a tail that extends towards frequencies that are *lower* than the normal frequency of the line. This tail is due to random motions, and from its size we can again derive the distribution of these motions. In such a manner η was found to be about 8.5 km/sec, a value that corresponds well enough with what had been found from the other evidence.

With radio telescopes of better resolving power it will be possible to resolve also the 21-cm line into individual components, and to derive a much more detailed picture of the distribution of the velocities as well as of the sizes of interstellar clouds. In particular it will be possible from these data to obtain statistics on the frequency and size of the large agglomerations of clouds.

The 21-cm measures have already enabled us to derive two other quantities that could not be well determined from other data, namely the average density and the average temperature of the interstellar gas. For the first we found 1.4 hydrogen atoms per cm³ (in the spiral arms), while the temperature was found to be about 125°.

Finally, attention should be ...

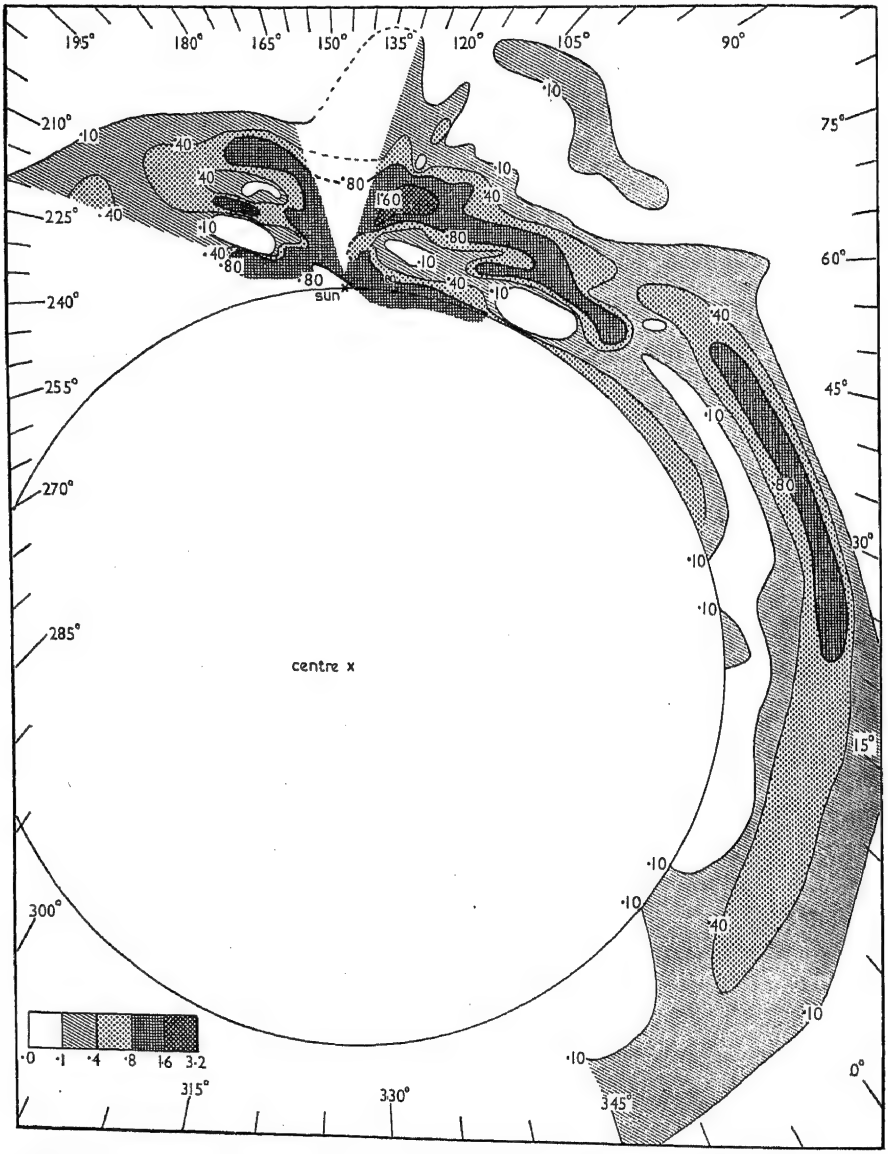

Fig. 1. Large-scale distribution of atomic Hydrogen in the outer parts of the Galaxy (from B.A.N. 452).

the interstellar gas. The profiles of the 21-cm line observed in various longitudes show pronounced humps at various velocities. These are due to large-scale unevenness of the distribution of the hydrogen. If we know the way in which the rotation of the Galactic System varies with

the distance R from the centre, and if in addition we know the distribution function of the random motions, it is possible to transform the line profile giving the intensity as a function of the frequency, into a curve giving the density of hydrogen as a function of the distance from the sun. In this way it was found that the hydrogen (and probably all the interstellar matter) is concentrated in long "arms", which are presumably wound spirally around the centre of the system, like we observe them in the spiral nebulae. Fig. 1 shows the general distribution of hydrogen in the outer part of the Galactic System. The shadings represent the different densities in the galactic plane. The attached numbers are numbers of hydrogen atoms per cm^3. The numbers just inside the frame are galactic longitudes. The sector from 220° to 320°, which is invisible from our latitude, has not yet been observed. No data relating to the inner region of the System are shown, as these are still too incompletely known.

The arms are probably mainly made up of gas, but are embedded in a medium of stars with a rather larger overall density. While the gas density in the arms is about 2.5×10^{-24} g/cm^3, the average overall density of gas + stars in the vicinity of the sun is about 5.6×10^{-24}. The gravitational potential field of the Galaxy is principally due to stellar masses, the gas contributing only a small fraction.

The arms are flat structures, the thickness perpendicular to the galactic plane between "half-density" points being about 250 parsec. The corresponding width in the galactic plane is about 3 times larger. The space between the arms appears to be practically devoid of gas. Like in most spiral nebulae the arms in the Galaxy are irregular. So far as one can judge from the parts of arms disclosed so far, the direction of winding compared to that of the rotation is such that the arms are trailing (in Fig. 1 the direction of rotation is clockwise). In general, however, the arms deviate but little from circles.

Over the entire region of the Galaxy surveyed, including the inner parts, the gas clouds are closely confined to the same plane, and form everywhere a layer that is extremely thin in comparison to its total extent in the plane. The average motion appears everywhere to be practically circular around the galactic centre. The angular velocity of rotation increases towards the centre. Fig. 2 shows the variation of the linear velocity with distance from the centre R in kiloparsec. Division by R gives the corresponding angular velocity in units of 1 km/sec.kps or 3.24×10^{-17} radians/sec.

It should be noted that over the whole of the Galactic System the

variation in angular velocity is quite considerable. Near the sun, for instance, the variation in ω if R is increased by one kps is 0.40 radians per 10^8 years. As the time of revolution is about 230 million years it is

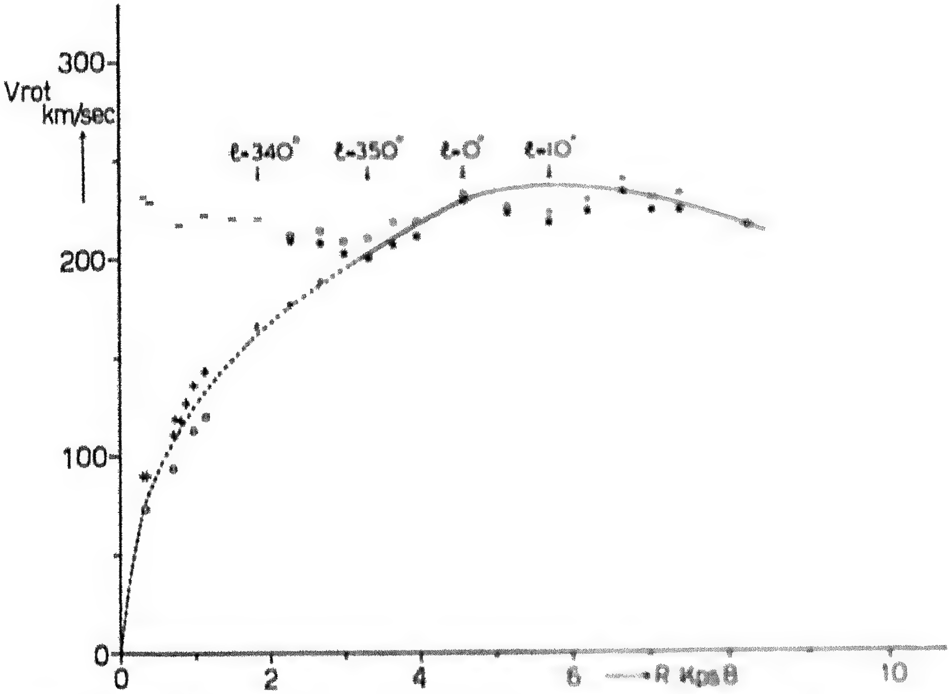

Fig. 2. Linear Velocity of Rotation in the Galaxy (for somewhat newer data and a detailed explanation, see B.A.N. 458).

clear that all structural details will be very much drawn out already in one revolution of the system.

Recent measures of the 21-cm radiation in the innermost region of the Galactic System, within 3 kps from the centre, show that in this part the irregular motions in the gas become much larger than near the sun [4]. They are estimated to be of the order of 50 to 70 km sec in the region between 1.5 and 2.5 kps from the centre, and become still considerably higher in the region within 1.5 kps. This indicates that the motion of the gas in these regions may be governed by a mechanism differing from that which causes the random motions in the outer parts of the Galaxy.

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DISCUSSION

BATCHELOR: How is it explained that the angular velocity ω is constant near the centre of the Galaxy and decreases like $1/r$ near the edge?

OORT: The gravitational field in the Galaxy is determined mainly by the distribution of stars. In the very central part we have roughly: force $\propto$ inner mass $\cdot\, r^{-2} \propto r^3 \cdot r^{-2} \propto r$. Assuming stationary circular orbits we then find $\omega =$ constant. In the outer parts the motion is more like Kepler orbits with $\omega \propto r^{-3/2}$.

VON KARMAN: Is it permitted to assume stationary orbits or will the motion be slowed down?

OORT: At the present rate of rotation the sun must have performed about 20 revolutions around the centre in the time of 4×10^9 years. There seems no reason for appreciable changes in the rotational velocity during that time. The inner part has performed many more revolutions and may have been affected by some hydrodynamical effects.

BONDI: We must be careful in trying to derive the mass distribution from the observed law of $\omega(r)$, as is done for other galaxies, as the mass occurs in an integral equation and such equations can give results depending very critically on the input functions $\omega(r)$.

SCHATZMAN: The increase of ωr^2 with increasing r will according to the work of Taylor and Jeffreys make the outer parts stable against radial convective transfer of momentum. So the non-uniformity of the rotation probably cannot be considered as a cause of turbulence.

BONDI: Would the fact that Morgan's spiral arms show a larger inclination than those from the 21-cm line lessen the problem of the age of these arms?

OORT: No, they would be stretched out and vanish in a time of the same order, say, about 10^8 years.

SCHATZMAN: The fact that the associations of O stars, that are

thought to be formed in spiral arms, are found to be very young, agrees with this age estimate.

OORT: It would indeed be a crucial test to find observational evidence for young and old spiral arms to exist side by side in our Galaxy.

MCCREA: Does the low emission of 21 cm radiation between arms necessarily mean emptiness or may all hydrogen be molecular in those regions, e.g. because of a low temperature?

GOLD: A low temperature in itself would also be sufficient to account for an absence of radiation, would it not?

OORT: The observations of the 21-cm emission line can show only the atomic hydrogen. So far as these observations are concerned, inter-arm space might be filled with molecular hydrogen, or with atomic hydrogen at a temperature considerably below that found in the cool clouds in the arms. But this hypothesis appears unlikely on the ground that, in the Andromeda nebula at least, the dust appears to be concentrated in the arms. It would also be difficult to understand why on this picture the massive young stars would exclusively be formed in the spiral arms, where we now observe them, and not equally often in the regions in between.

CHAPTER 5

PHOTOGRAPHIC STUDIES OF SOUTHERN EMISSION NEBULAE

BY

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During the past two years, the Armagh-Dunsink-Harvard Telescope of Baker-Schmidt design and the *f*/1.5 Zeiss-Sonnar camera, on loan from Mr. Richard S. Perkin, have been used extensively at Boyden Station for the photography of Hα-emission features of the southern Milky Way. The two papers that follow, by Dorrit Hoffleit and by Bok and Wade, report on those results obtained to date that should be of special interest for studies of the gas dynamics of interstellar matter.

The Hoffleit paper deals with a survey of the Carina section of the Milky Way. This section is traditionally known as one of the richest in O and B supergiant stars and associated emission nebulosity and therefore merits special study. It constitutes the greatest near-by concentration in the known spiral arms of the Milky Way and in this section of the Milky Way we apparently have our very best opportunity to study problems of evolution of supergiants and of the interaction of interstellar gas and dust with recently-born stars. Several features are noted specifically by Dr. Hoffleit, but the one that is probably most significant from an evolutionary point of view is the presence of numerous small, roundish, dark "globules" seen projected against — and presumably at the periphery of — the conspicuous emission nebulae excited by groups of stars with a plentiful supply of ultraviolet radiation. This phenomenon, which is most striking for the Carina section, has also been noted for at least two other emission nebulae, Messier 8 [1] and the nebulae NGC 2237, 2238, 2246 associated with the cluster (or association) NGC 2244 [2]. It is indeed surprising that the small, dark globules should occur at the edges of H II regions and that they are not seen projected against reflection nebulae, such as the Pleiades nebula. Some of the larger roundish, dark nebulae are found associated with the large dark cloud complexes in Taurus, Auriga and Ophiuchus, but the smaller ones seem

to favor the outer boundaries of emission nebulae of relatively high excitation. It does not seem likely that the explanation of the observed preference can be that the small dark globules require a good luminous background in order to be visible at all, for the reflection nebulae and emission nebulae of low excitation would seem to provide just as good a background as the emission nebulae of high excitation.

We should warn students of cosmical gas dynamics that the Carina section of the Milky Way cannot be considered average or typical. The distribution of the emitting gas is much more irregular along the Milky Way than is the distribution of O- and B-stars or of cosmic dust. Some of the problems of the irregular relative distribution of interstellar gas and dust are discussed in another note in the present volume (see Ch. 41) and the contrast between the Carina and Sagittarius sections is analyzed in a paper in press for the forthcoming volume *Vistas in Astronomy*.

The note by myself and Campbell M. Wade, about a classification system of $H\alpha$-emission features, is a by-product of an extensive study of southern emission nebulae, made in an attempt to trace the course of one or more sections of spiral arms of the southern Milky Way. A complete list of southern $H\alpha$ emission nebulae together with an analysis of their distribution is now being prepared for publication elsewhere (a joint paper by Bok, Wade, and Michiel J. Bester of the Boyden Station). The participants of the Symposium are presumably most interested in the variety of structures observed. In some sections of the Milky Way the structure of the emission nebulosity is highly turbulent, whereas relatively smooth features may be found only 10° to 15° from the turbulent section. We note in conclusion that on page 228 we report on a related study by Wade and myself; this study deals with the variation along the band of the Milky Way of the percentages of O- and B-stars with and without associated emission nebulosity.

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Observational Features in Carina of Interest for Dynamical Theories of Interstellar Clouds and Stellar Evolution

by

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That our galaxy, the Milky Way system, is spiral-shaped somewhat like the Andromeda Nebula has long been accepted as fact. The sun's location within the system has, however, heretofore made an analysis of the detailed structure of our system difficult. But within recent years investigations particularly by W. W. Morgan and by W. Baade have yielded significant progress in defining sections of two spiral arms accessible to study with northern instruments; and definitely confirmed which types of celestial objects are specifically associated with spiral structure and are not found either between spiral arms or at great distances from the galactic plane. Most definitive of such constituents of spiral arms are the diffuse emission nebulosities and high-luminosity early-type stars (those with spectral classes O and B). Probably the richest region in the whole sky for emission nebulosity and B-type stars is the region of the η Carinae nebula in the southern hemisphere. The η Carinae nebula itself, constituting a "knot" in a spiral arm, is some 2° in diameter in which over a hundred B-type stars are known to be embedded and more are likely to be discovered as our instruments reach stars of fainter apparent magnitude, and larger-scale instruments permit presently overlapping spectra to be clearly separated.

A preliminary survey has recently been completed of the bright nebulae in a 75 square degree area, 5° wide by 15° along the galactic circle from Vela to Centaurus, centered on η Carinae[1]. Both red and blue sensitive plates were used, taken with the 32–36-inch Baker-Schmidt type telescope at the Boyden Station of Harvard Observatory. The photographs show stars to about the 19th magnitude (photographic) and a diversity of nebular detail not previously recognized in the region. Within the 75 square degree area of the survey, 70 separate nebulosities or distinctive satellites of larger formations have been catalogued. Of these 30 are diffuse and irregular while some 40 are small, round, and fairly regular in shape. The distributions of the early-type stars with respect to the various nebulosities have been investigated. From the B-type stars apparently involved in them, the distances of the nebulosi-

Fig. 1. Composite from ADH Plates of the Carina region taken at the Boyden Station of Harvard Observatory, Bloemfontein, South Africa. All prints North at top, East at right. Prints 1 and 2 enlarged 2 ×, all others from 4 × to 6 ×. This is a negative print in order to show better the faint emission nebulosity.

Picture 1. One of the most conspicuous and complex groups of nebulosities separated from the large η Carinae nebula nearly 4° to the East. At least ten small dark nebulae or globules are superposed on each of the two large bright nebulae. The two most prominent bright condensations are separated by 0°.4 or about 10 parsecs. The letters *a*, *b*, and *c* mark three small nebulae. Nebula *a* is red and the order of 0.7 parsec in diameter. Nebula *b* is spiral-shaped and strong in $H\alpha$. Its longest dimension is about one parsec. Nebula *c* is more compact and has a higher surface brightness than either *a* or *b*, but is less than a light year in apparent extent. At *d* a typical globule (with star superposed) is marked. It is only half a light year across.

Picture 2. A "Rosette Nebula", 3° West of η Carinae. The cluster IC 2581 is in the somewhat separated left segment of the bright nucleus. A few prominent globules are seen superposed on the separated arcs of bright nebulosity at the right. At the lower left corner a red nebula, diameter about 3 parsecs, surrounds the 8th-mag. K2 supergiant, HD 90289.

Picture 3. Planetary, diameter 35″ or under one light year.

Picture 4. Planetary, diameter 30″ or 0.6 light year.

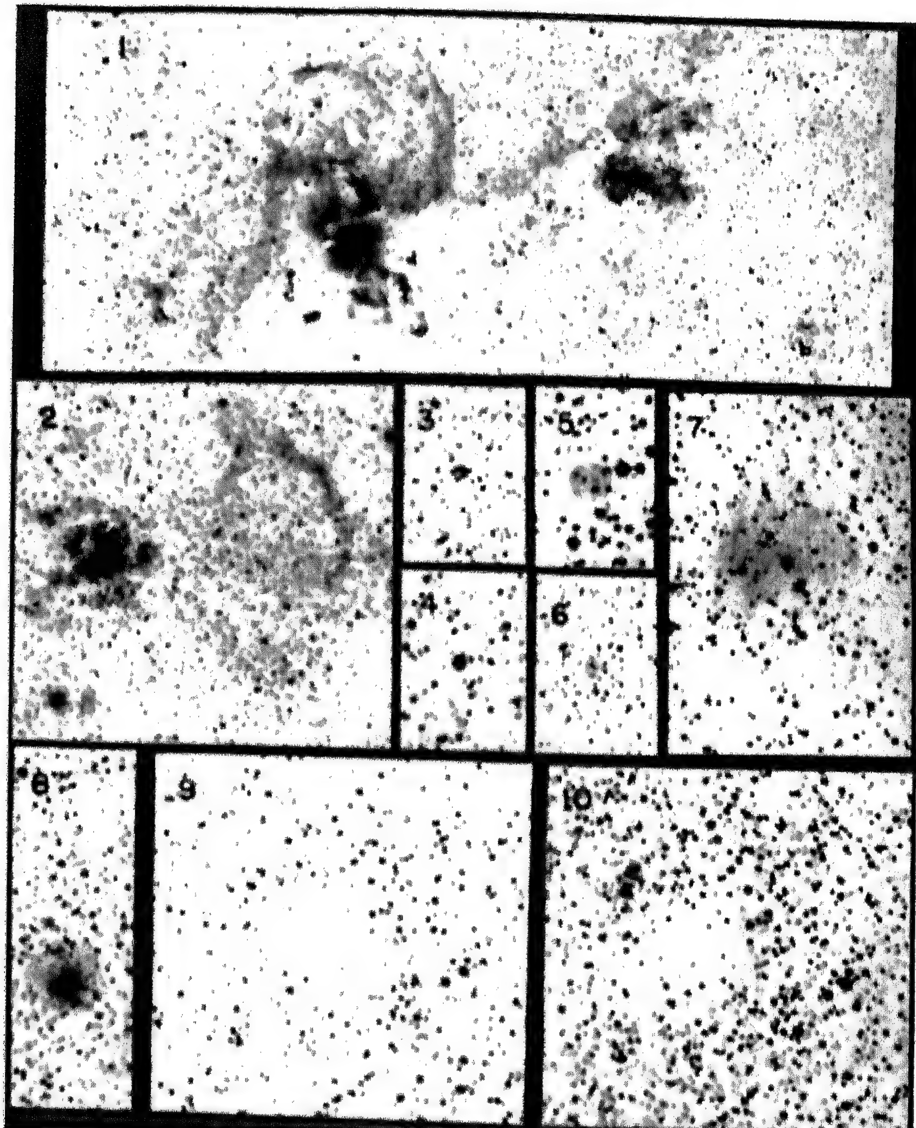
Picture 5. Neutral color, diameter 70″ or 2 light years.

Picture 6. Faint, red, diameter 60″ or about 1.5 light year.

Picture 7. Centered on 9th-mag. B8 star, strong in $H\alpha$, diameter 4′ or 1.7 parsec.

Picture 8. Centered on 9th-mag. M0 star, diameter 100″ or 2.5 light year.

Pictures 9 and 10. Most perfect example of "Löcher" on red and blue plates, respectively. Diameter 3′ or 1.5 parsec.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ties have been estimated. For the most part, the nebulosities probably are 1300 to 1500 parsec distant. The thickness of the spiral arm in this region is the order of 400 to 500 parsec. Dimensions of the various types of nebulae have been estimated on the basis of a common distance of 1500 parsec. The presently available accuracies in both the estimated individual distances and in the apparent diameters of diffuse objects do not yet warrant the use of other than a common order-of-magnitude distance for linear size-determination.

Numerous of the observational features noted during the survey should have a bearing on any dynamical theories relating to the structure of interstellar clouds and to stellar evolution. These features fall roughly into seven categories:

(1) *Irregular diffuse areas of Hα-emission,* some completely amorphous, others showing characteristic structural details. Diameters from under a parsec to more than 50 parsec.

(2) *Short arcs of bright nebulosity,* some isolated, others constituting details of larger formations, many of which have the same radius of curvature (about one parsec) and show a tendency toward a preferred orientation in space.

(3) *Small, fairly regular, round or spiral-shaped nebulae* with diameters from 0.5 to 3 parsec, most of them showing a central star.

(4) *Globules*—small round dark nebulae with diameters usually under 0.3 parsec and probably averaging 0.1 parsec.

(5) "*Löcher*"—small roundish dark nebulae, larger than globules and usually with less sharply defined boundaries, seen superposed on rich star-fields or bright nebulosities. Diameters the order of a few parsec.

(6) *The concentration of early B-type stars* with respect to emission or filamentary absorption nebulae. There are some 50 per square degree in the densest parts of the η Carinae nebula in contrast to 10 per square degree for the area as a whole. In some Sagittarius regions briefly examined, an area of much filamentary dark nebulosity shows 6 B-stars per square degree while nearby regions with no filamentary nebulosity average less than one B-star per square degree.

(7) *Indicated correlations between intrinsic luminosity and color excess* for B-type stars. The intrinsically most luminous B-stars in Carina appear strongly reddened in comparison with those of lower luminosity.

The diversity of appearance of the nebular formations is striking. The comparatively large η Carinae nebula shows obvious evidence for turbulence. Much of it appears to consist of overlapping short arcs of nebulosity of nearly uniform radius of curvature (about one parsec). While examples can be found of the orientation of such arcs in almost any direction, it is remarkable that there is a preponderance of crescents concave more or less toward the north galactic pole. In the overall area investigated, arcs of larger radii of curvature, up to about 10 parsec, are found. Some of these, on closer inspection, are seen to be envelopes of arcs with smaller radii. The central portions of the η Carinae nebula are too dense to reveal any such detailed structure. The nebula is trifurcated in appearance by lanes or wedges of dark absorption material. In the near outlying regions of this nebula, "fronts" of bright nebular material, some consisting of envelopes of small arcs are found. The whole area within 2°–3° of the center gives the impression of a large diffuse mass in which both central forces and a uni-directional outside force might be operating.

In the more distant outlying regions, other types of formations are found, and arcs when present show less tendency toward any particular preferential orientation. In some cases the lines of bisection of the cusps are directed toward an open cluster, in some others, in the general direction of η Carinae. The open cluster IC 2581 is a striking example of an open cluster and a dense small knot of nebulosity (2 parsec in diameter at 3° west of η Carinae) partially encircled at a distance of about 7 parsec by an envelope of arcs whose bisectors are clearly directed toward the nucleus.

In contrast to the turbulent-looking η Carinae nebula, IC 2944, 7° to the east shows completely different characteristics. The nebulosity seems thinner (more transparent) and reveals very little evidence of systematic structure. The clarity with which dark globules stand out against this bright nebula suggests that its relatively quiescent state may be favorable for the accretion of the dark matter into globules or primordial stars.

In a private communication Dr. Uco van Wijk, who has been studying microphotometer tracings of globules in the Carina nebula, comments that he finds globules with indications of what appear to be emission rims. He also reports that the red absorption is stronger than the blue, indicating that the globules are actually embedded in the nebula.

The 40-odd small round bright nebular found in the Carina survey represent a variety in size, color, and the spectral class of the central

star. Seven of the nebulae are very small, only about 0.2 parsec across, and appear to be ordinary planetary nebulae. The others are larger, ranging from 0.5 to 3 parsec in diameter. Twenty-one are stronger in red light, nine in blue, while eight are about equally intense on the two kinds of photographic emulsions used in this survey. The spectral classes of the central stars are available in ten cases, six for red nebulae, four for blue. The spectral classes for the blue are all B, B 3 and B 5. As the energies emitted by such stars are not likely to be sufficient to excite blue nebulosity, it may safely be assumed that the blue nebulae are of the reflection type. The spectral classes of the central stars of the red nebulae range from B0 through M0, with only F and G-types not represented. Most of the stars are presumably high-luminosity stars and the nebulae probably include examples of both reflection and emission.

Several investigations of a few years ago, inspired primarily by Bok, led to the theory that globules are primordial stars which eventually collapse into high-luminosity early-type stars. This theory appears to be supported by the prevalence of both globules and early-type stars in the same parts of the Milky Way. It is suggested here that the small bright round nebulae may represent an intermediate stage between the globules and the "finished product" high-luminosity early-type stars. Such an intermediate stage would presumably be of short duration in comparison with either the globule or the B-star stage. The relative numbers of the three types of objects found in Carina, namely 150 globules, 30 to 40 small round bright nebulae, and 780 B-type stars, would lend support to the hypothesis that the round nebulae are relatively short-lived.

A comparison of available data on absolute magnitudes of B-type stars in the Carina region with photoelectric colors and color-excesses determined by Bok and Van Wijk shows a pronounced correlation, the stars of the highest luminosity having color indices of about $0^m.3$ greater than those of the lowest luminosity group. Normal colors for stars of the spectral and luminosity classes represented are not expected to differ by more than about $0^m.06$. (The correlation found is based on few stars and needs to be thoroughly verified.) The implied difference in total absorption for the high- and low-luminosity groups amounts to nearly two magnitudes. This is tentatively attributed to a residual shell of globule dust still surrounding the new-born high-luminosity stars. This interpretation would seem in keeping with the proposed hypothesis to account for the small round nebulae.

To summarize, then, the preliminary survey of the nebulae and their stellar [illegible]

dynamics of interstellar clouds should take into consideration the following observed phenomena:

(1) *Turbulence*

(a) That turbulent swirls are strongly evident in the η Carinae nebula but almost completely lacking in the nearby nebula, IC 2944.

(b) The preponderance of arcs whose bisectors are oriented toward the general direction of the north galactic pole, toward the central η Carinae nebula or toward an open cluster.

(c) The uniformity in apparent radii of curvature of a great many of the arcs.

(2) The high *concentration of* B-*type stars* where the emission is most intense (50 per square degree in the northern sector of the η Carinae nebula in contrast to 10 per square degree for the area as a whole).

(3) The *range in color* of the small round nebulae and the diversity in spectral class of their central stars.

(4) The *occurrence of globules and Löcher* and their probable structure.

(5) The *high color excess of the supergiants* in comparison with the less luminous B-stars, suggesting shells of cosmic dust around the luminous supergiants.

(6) The *progression of relative sizes* of the various types of bright and dark objects that abound in this region: B-stars, globules (order of 0.1 parsec in diameter), Löcher (1 to 3 parsec), planetaries (less than 0.3 parsec), small round bright nebulae (0.5 to 3 parsec), irregular diffuse nebulae (1 to 50 parsec), and the larger Hα-regions studied on small-scale plates by Bok, Bester and Wade. How are these various constituents of the spiral knot related to such physical parameters as turbulence, mass, density and age; and can the majority of them be arranged in a logical evolutionary sequence?

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Fig. 2. Nebula NGC 6188; the dark area right of the luminous rim is covered with Hα-emission on longer exposures. Photographed with the 60″ Rockefeller reflector at the Boyden station. Scale 1 mm = 14″.3 (1.8 × original size).

Fig. 3. Nebula IC 2944 and globules; smaller single ones with diameters of 0.2 light year, others up to about one light year. "Front" of arcs to west has weak emission on one side, absorption on the other. Enlargement 7 × from a photograph taken with the Armagh-Dunsink-Harvard 32-36-inch Baker-Schmidt telescope at Harvard Observatory's Boyden Station, Bloemfontein, South Africa. North at top, West at right, scale of reproduction 1 mm = 9″.7.

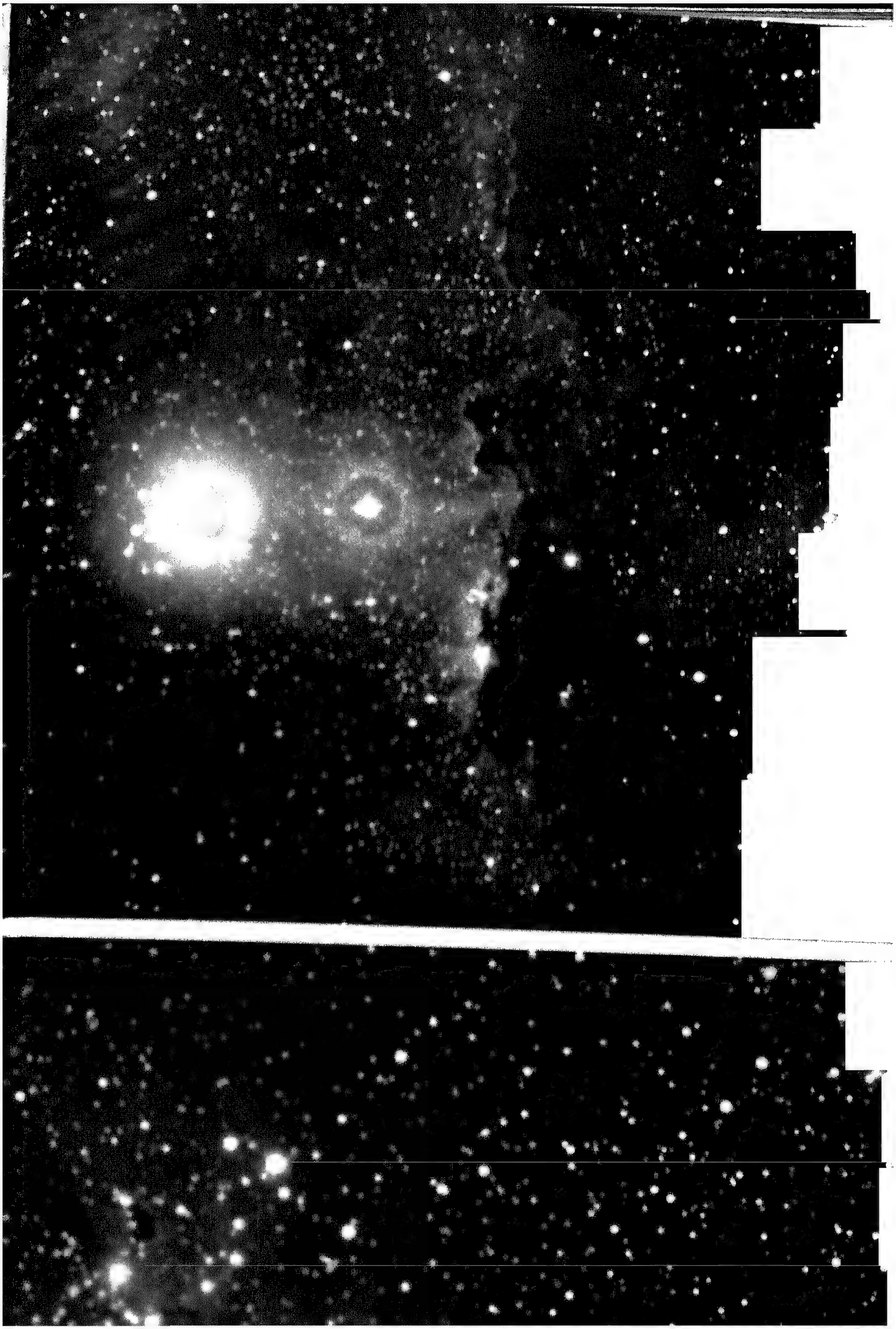

Fig. 4. Region of NGC 6334; the concentrations shown here are the bright knots shown on small-scale photographs. The lower right condensation is NGC 6334. The other nebulae have no NGC number. Photographed with the 60-inch Rockefeller reflector of the Boyden station, no filter. Scale 1 mm = [illegible]

A PRELIMINARY CLASSIFICATION SYSTEM FOR Hα-EMISSION NEBULAE

by

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The classification system described in the present note has been developed in the course of the past year to provide a concise qualitative description of the Hα-emission nebulae located on photographs made with the Zeiss camera (PZ) at the Boyden Station [1]. The classification is by shape, structural features and surface brightness. The apparent and linear size of the nebula and the spectral type and absolute luminosity of the exciting star do not enter into the classification.

Type I: M 8, the Lagoon Nebula in Sagittarius, is representative of Type I; the nearby Trifid Nebula, M 20, is another example of this type; so is the central part of the η Carinae Complex. Type I has the greatest observed surface brightness. If we assume total absorption at the outer edge of the nebula of all ultraviolet radiation beyond the Lyman limit, then the "Emission Measure" as defined by Strömgren [2] (the emission measure equals the product of N_H^2, the number of H atoms per cm^3 squared, times the length of the emission path in parsec) amounts to 15,000 or more. Diameters are of the order of 10 to 50 parsec. The presence of associated globules and irregular dark material is characteristic of Type I. N_H appears to be of the order of 20 to 200 H atoms per cm^3.

Type II: This type is faint to moderately bright, irregularly distributed, and frequently shows a marked filamentary structure. Type II is generally faint, with Emission Measures of the order of 500 to a few thousand, $N_H = 15$ or less, diameters 10 to 50 parsec. Example: NGC 6604. The most striking example of filamentary type II nebulosity in the southern hemisphere shows strongest evidence of turbulence at the position of the Vela extended radio source [3]. The smallest filaments shown on the coarse PZ plates measure 8 parsec in length by 0.1 parsec in thickness. The Emission Measure is of the order of 2000, $N_H \sim 5$ to 10, and the diameter of the entire complex is of the order of 50 parsec. Another striking example occurs near IC 4628.

Type III: The ideal type of spherical Hα nebula, with the exciting star near the geometrical center; Type III is the closest observed ap-

proach to one of Strömgren's H II spheres. The nebula surrounding NGC 6383 is representative of Type III. The Emission Measure is of the order of 5000, $N_H \sim 10$, and the diameters vary between 10 and 40 parsec.

Type IV: A ring structure—or an incomplete ring—is characteristic for Type IV. This variety of nebula seems very much like the partial rings shown by the emission nebulae found on the Mount Wilson and Palomar Hα-photographs of the spiral arms in the Andromeda Nebula. Examples: NGC 2327 (near Sirius) and NGC 3581 (near the η Carinae Nebula). The Emission Measure is of the order of 1000 to 10000, $N_H \sim 10$ (or higher in the denser portions) and the radii of the rings are of the order of 5 to 25 parsec.

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DISCUSSION

MINKOWSKI: (1) The suggestion that the radio source in Puppis is connected with the filamentary nebulosity shown by Bok is not correct. Its position coincides exactly with an inconspicuous nebulosity that resembles the Cassiopeia source.

(2) Why are not the disk-type objects called planetary nebulae by Dr. Bok? With his powerful instrument he should find many planetaries.

BOK: Dr. Hoffleit suggests in her paper that the sizes and spectra of stars caution against calling all of them planetaries. The average linear diameters for these objects are three times as great as those of normal planetaries. Also, some of the apparent central stars are too red to be considered likely nuclei of planetary nebulae.

LAPORTE: Could you tell us something about the star η Carinae?

BOK: This would lead us too far afield, for η Carinae is a very peculiar star. It is not certain that the nebula is excited by η Carinae itself, for there are very many O and B supergiants in this part of the sky.

CHAPTER 6

CERTAIN PECULIAR STRUCTURES IN INTERSTELLAR CLOUDS

BY

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Simeis, Crimea, U.S.S.R.
(Communicated by letter of 10 June 1953)

(1) We have found here two unique nebulosities, which may be considered as peculiar hydrodynamic formations*. In the light of the ideas expressed by Burgers and other hydrodynamicists, they may be considered as highly stretched genuine "cosmical gas streams": one of them is in Cygnus about 6–7° long, and the other in Perseus about 5° long (the latter is NGC 1499 with an unknown prolongation, as long as NGC 1499 itself). Both nebulosities are partly of filamentary nature. Some discussion of this phenomenon, as well as photographs of these nebulae are given in a recent paper by G. Shajn and V. Hase in the "Astronomical Journal" (Russian) **30**, 130 (1953). A better reproduction of the first nebulosity may be found in the "Atlas of Diffuse Gaseous Nebulae of the Crimean Astrophysical Observatory" (1952), photo No. 24.

(2) From the aerodynamical point of view [the hypothesis of collisions (Oort), shock waves (Burgers)] may deserve attention some new illustrations in two other papers by the same authors in the same number of the Astronomical Journal: "On the connection of the filamentary structure of nebulae with motion" (p. 125) and "Diffuse nebulae with some concentration of gas to the periphery and their interpretation" (p. 135). In particular it is proved that the striking filamentary nebulae discovered by us in Auriga turn out to form a nearly closed composite loop, probably not less instructive than the Network Nebulae in Cygnus.

(3) In connection with Zanstra's manuscript to be discussed at the Symposium "Formation of Condensations..." (with application to the

* Added in proof. Since then a number of other nebulosities of such kind have been found; see, for instance, the nebulosity in the [illegible]

planetary NGC 7293). I would like to call attention to the presence of a filamentary arc at some 14′ from the centre of the nebula, which arc, I think, was not known previously. The photograph of NGC 7293 and some discussion is given in the first of the three papers in the "Astronomical Journal, 30, Nr. 2, 1953.

(4) Some new illustrations of tiny wavy structure, probably connected with the shock wave phenomenon, may be found in the mentioned Atlas of diffuse nebulae, particularly in the region near the Pelican nebula (Photo Nr. 34).

(5) Attention may be called to the investigation by S. Pikelner on the magnetic fields and the kinematics of interstellar gas between clouds [Dokladi Acad. Sci. of the USSR 88, Nr. 4, 299 (1953)].

CHAPTER 7

CHARACTERISTICS OF SOME DIFFUSE NEBULAE IN THE MAGELLANIC CLOUDS

BY

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(Communicated by letter of 21 May 1953)

The Large Magellanic Cloud is noted for its richness in diffuse nebulae most of which are highly irregular in outline. Like their counterparts in our own galaxy, they are usually involved in clusters of early-type stars which must be largely responsible for exciting the interstellar gases. While a certain amount of irregularity may be expected from the distribution of nearby hot stars there are many instances where a simple picture of overlapping H II regions will not suffice, and it is necessary to invoke varying densities of the interstellar gases.

The brightest nebula of all, that surrounding 30 Doradus, is characterised by an extremely complex pattern of loops. The purpose of this note is to draw attention to the fact that several nebulae appear in the clouds in which the nebulae, or even the involved stars, follow a relatively simple looped pattern—that of a letter *S*. There are at least three cases where the observations strongly indicate a real distribution of gas in this form, and not a spurious effect induced by patchy absorption or radiation.

(1) Nebula near S Doradus. This is the finest example of a pure *S*, outlined almost entirely by nebulosity. At the centre there is a cluster of moderately bright blue stars, but the brightest stars in the neighbourhood are S Doradus within the preceding loop and a very compact cluster 2′ of arc north; these and other bright blue stars show no sign of nebulosity in their immediate neighbourhood. The rich field of stars, close to the main axis of the cloud, carries no suggestion of patchy absorption. It would be difficult to account for this nebula without assuming that it represents a region of unusually high density of interstellar gas illuminated by the nearby hot stars. The nebula is about 8′ of arc across, corresponding to about 100 parsec.

(2) NGC 346. This is the only nebula in this series in the Small Cloud. A dense elongated cluster of stars marks the axis of a reversed *S* whose ends are chiefly defined by nebulosity. One of the brightest regions of nebulosity consists of a curved wisp at the north preceding end of the axis.

(3) NGC 2074. This nebula is very similar to NGC 346 in that a reversed *S* is defined along its axis by bright stars and by nebulosity, and at its two ends chiefly by nebulosity.

(4) NGC 1763. A region extremely rich in nebulosity of which the brightest, NGC 1763, suggests the form of a direct *S* except that the northern bay is largely filled in by luminous gas. As in the three preceding cases there is a cluster of bright stars at the centre, which shows signs of elongation along the axis of the S. Note the loose cluster south of NGC 1763 apparently quite free from involved nebulosity. North following NGC 1763 are two "stars" HDE 268721 and 268726 classified by Harvard as B-type. Both appear nebulous on Radcliffe photographs, 268726 having a nebulous halo about 14″ of arc across and a predominantly nebular spectrum.

(5) NGC 2078, 2083, 2084. A direct *S* which is apparently defined mostly by nebulosity but also by moderately faint involved stars. In this instance it should be noted that the bay of the *S* might be produced by a tongue of dark absorbing matter.

(6) HDE 269730 (not in NGC). A cluster of bright stars embedded in faint nebulosity, brightest at its south following end where a circular loop is suggested. In this instance the *S* form seems to be outlined by the bright stars as well as by the nebulosity.

(7) NGC 1872 etc. A region of very faint nebulosity with involved stars. The brightest stars appear to take the form of a direct *S*, the northern bay being larger. The impression is very much stronger on small-scale photographs, and it is probable that the effect would be enhanced on this large-scale with a longer exposure.

These photographs pose the problem:—are the spatial shapes of these nebulosities really in the form of *S*, being concentrated mainly in one plane, or do they represent projections of a more complicated 3-dimensional pattern, e.g. a helix? A single turn of a helix viewed from various angles could appear as

and with all possible orientations the last two should be the commonest types. These latter types have not been observed. Although the sample

of nebulae presented here is a very small one, the observed ratios of length to breadth are roughly in accordance with statistical expectation if these nebulae are concentrated mainly in randomly orientated planes, in the form of the letter *S*.

It may be pointed out that the shape of the Large Cloud itself, as shown on small-scale photographs, is very roughly of *S* shape.

The writer is not aware of any examples of a similar appearance among diffuse nebulae in our own galaxy, although there is one very familiar instance among the dark nebulae (Barnard Area 75, near θ Ophiuchi).

These examples from the Large Cloud show no preferential distribution with respect to the main axis of the Cloud and probably indicate some special local conditions. The irregularity of the Cloud implies that the effects of a central gravitational force and of differential rotation must be far less marked than in our own galaxy.

CHAPTER 8

A SURVEY OF PROBLEMS AND SUGGESTED SOLUTIONS

BY

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The following survey of problems and suggested solutions was prepared as one of the preliminary communications of the Symposium. The observational data reviewed in the preceding chapters present an overwhelming multitude of details. From these have been singled out a number of well-defined problems that can give rise to theoretical considerations. The problems (1–6) and suggested solutions (A–F) are put somewhat in their historical context. Critical arguments to show which problem is most important and which solutions correct have not, in general, been included. The text is mostly unchanged, except for references to the discussions, that have been added afterwards.

(1a) THERE IS INTERSTELLAR GAS. WHY?

(1b) THERE ARE YOUNG, HOT STARS. WHY?

The problem of stellar evolution, of which (1b) is a part, is *not* included in the subject matter of the symposium. But it is important in connection with (1a) and also with C, below. It is certain that hot stars are born from interstellar matter. Question (1a) can then be framed: why is not all gas condensed into stars?

Tentative answer: there is a balance, gas being used up by condensation into stars and by accretion, and gas being replenished by various processes from stars.

This raises a number of difficult questions discussed in more detail by Biermann in Ch. 39.

(2) WHAT IS THE ROLE OF DARK MATTER?

It is mainly a nuisance for galactic research and its direct dynamical effects are almost nil, although earlier researches have made an important point of the radiation pressure on the grains. Yet possibly important as a cooling agent for the gas and as an absorber of Lyman alpha quanta

and thus indirectly for dynamics. Further it is important as a tracer of
(a) structural details of gas clouds, for dust and gas go mostly together (the various arguments pro and contra this assertion are discussed in Chs. 40 and 41);
(b) magnetic fields, if a magnetic theory of interstellar polarization holds, or
(c) relative motions of dust and gas, if a wind theory of interstellar polarization holds (Gold).

(3a) WHY DO INTERSTELLAR CLOUDS EXIST?

(3b) WHY DO INTERSTELLAR CLOUDS MOVE?

The interstellar gas has local regions of high density (clouds) in a wide variety of sizes, often with well-defined boundaries. The motions of some are fairly well known. Why are not the clouds and their motions gradually washed out by encounters and viscosity? This was a central subject at the Paris symposium and it is again at the present symposium. Suggested answers to either or both questions are:

A. *Gravitational instability.* This theory is the oldest one (Jeans). The Paris discussions and estimates made by Chandrasekhar (who worked out a theory including turbulence [1]) show that gravitation is unlikely to be important, except for small and very dense clouds.

B. *Turbulence and magnetic fields.* That both are likely to go together was discussed at length in Paris. Starting from the clear picture of turbulence in an incompressible homogeneous medium the question was asked: can the turbulent pressure differences give rise to "clouds" if effects of compressibility enter? There was a long discussion that may be continued now. (A further clarification was indeed reached but the emphasis has somewhat shifted. See Lighthill's Ch. 22 and the general conclusions in Chs. 42 and 43).

C. *Radiation from O-stars.* Another clearly understood picture is that of ionized spheres (Strömgren spheres) around hot stars in a homogeneous gas. There are theoretical and observational grounds for assuming a temperature fall from perhaps 10000° in the ionized regions to perhaps 100° in the neutral regions. At Paris Spitzer mentioned the tendency to equalize the pressures as a reason for cloud formation but it was neglected in the discussion. Subsequent thought by Spitzer, Oort and others has shown that it is likely to be of great importance. This idea combined with the event of the birth of an O-star may also give violent cloud motions and by these motions further compression (these problems are reviewed by Oort in Ch. 28).

D. *Spontaneous condensation.* Zanstra has suggested that the peculiar cooling mechanisms of gases illuminated by a hot star may in itself be sufficient to allow a dense phase and a tenuous phase of the gas to exist side by side. Any accidental cause might then lead to cloud formation (Ch. 13).

Any of these explanations have been suggested for objects varying widely in size. Of these we may mention

a)	The cometary condensations in the Aquila nebula	0.001 parsec
b)	Globules and detached patches of dark matter	0.5 ,,
c)	The clouds shown in Adams' interstellar lines	10 ,,
d)	The clouds that are seen as condensations in spiral arms	100 ,,
e)	The clouds from which spiral arms themselves originate	500 ,,

It will be an important task to find criteria indicating which explanation may be correct for which type of object.

(4) GASEOUS EMISSION NEBULAE NOT EXCITED BY HOT STARS

Apparently such nebulae exist; what is their source of excitation?

(5) VERY SHARP DETAILS IN GASEOUS EMISSION NEBULAE

These are often observed, like filaments and luminous edges. How are they formed?

E. *Shock waves.* A number of years ago Oort and Burgers explored the properties of shock waves in the interstellar gas. Thereby questions 4 and 5 were linked, for a shock wave causes both heating and compression. This idea, which has been discussed at Paris, should again be examined critically and compared with laboratory experiments. This type of explanation or related types of aerodynamical motion may be considered for

a. Filamentary expanding nebulae (Cygnus loop, nova shells)
b. Filamentary nebulae with violent internal motion (Cass. radio source, Vela nebula?)
c. Luminous rims in a mixed dark and bright diffuse nebula.

F. *Ionized edges.* A simpler alternative explanation for luminous rims (c), in which only question 5 has to be answered, is that they are the ionized edges of dense neutral regions. Along this line also an explanation for the 'droplets' and 'elephant trunks' may be suggested. (The

papers and discussions in Chs. 16 to 19 give a variety of useful suggestions on points E and F. They emphasize the enormous complexity of the phenomena).

(6) CIRRUS-STRUCTURE IN REFLECTION NEBULAE

This type of very fine structure shown most beautifully in the Pleiades has found no adequate explanation. The suggestion of a sliding collision of two clouds has been made but it should be further explored (this was not yet done).

REFERENCE

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PART II

PHYSICAL CONDITIONS OF INTERSTELLAR GAS

CHAPTER 9

THE ELECTRICAL CONDUCTIVITY OF THE INTERSTELLAR GAS

BY

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According to the program, I should speak on the influence of radiation, ionization, and electrical conductivity on gas dynamics and flow problems. Now, the next paper to be read will deal with the influence of radiation, as far as it determines the temperature of the interstellar gas; and Professor Oort and I will deal with some dynamical effects of radiation on Thursday morning. Therefore, I will concentrate on the question of electrical conductivity of the interstellar gas.

This would be an easy question, if there were no interstellar magnetic field. Everywhere in the interstellar gas, even in the regions of low ionization (H I-regions), the conductivity is determined by the encounters between oppositely charged particles, because of the long range of the Coulomb forces. Then the conductivity is nearly independent of the density and of the degree of ionization, and is a function of temperature only. Its minimum value in the cool H I-regions is of the order 10^{10} sec^{-1} (= e.s.u.). That is quite a high value, if one regards the large cross sections that are available to currents flowing in the interstellar space. The time scale for induction effects—e.g. for the spontaneous decay of a magnetic field of linear dimensions L—is given to order of magnitude by:

$$t = L^2\sigma/c^2 \approx 10^{28} \text{ sec},$$

with

$$L \approx 1 \text{ parsec} = 10^{18.5} \text{ cm}, \quad \sigma = 10^{10} \text{ sec}^{-1}, \quad c = 10^{10.5} \text{ cm/sec}.$$

We can therefore regard the conductivity as infinite everywhere in the interstellar gas, provided its value is not reduced by a magnetic field.

There are reasons, which I am not going to discuss now, to expect

interstellar magnetic fields of the order $10^{-7}...10^{-5}$ gauss. In such a magnetic field the charged particles follow in the mean rather closely the magnetic field lines, the Larmor frequency being very small compared to the collision frequency. Intuitively, one would conclude that there is a considerable reduction of the electric conductivity perpendicular to the magnetic field.

I will show that this conclusion is not adequate for most of the interesting problems. Generally speaking, it is rather dangerous in magneto-hydrodynamics to start from the trajectories of single particles and to deduce from them, more or less intuitively, the macroscopic behaviour of the plasma. Of course it is possible to do it correctly that way, but it is far easier to start with an overall momentum balance, which has to hold strictly. The equation of motion of a charged particle (say an electron) is

$$m_e \frac{\mathrm{d}\boldsymbol{v}_e}{\mathrm{d}t} = -e\left(\boldsymbol{E} + \frac{\boldsymbol{v}_e \times \boldsymbol{H}}{c}\right) \tag{1}$$

(m_e mass of an electron;—e charge and $\boldsymbol{v}_e$ velocity of it; c speed of light; $\boldsymbol{E}$, $\boldsymbol{H}$ electric and magnetic fields). The corresponding macroscopic equation for the whole ensemble is the following

$$\varrho_e \frac{\mathrm{d}\boldsymbol{V}_e}{\mathrm{d}t} = -e n_e \left(\boldsymbol{E} + \frac{\boldsymbol{V}_e \times \boldsymbol{H}}{c}\right) - \operatorname{grad} p_e \tag{2}$$

(ϱ_e, n_e mass and number density; p_e pressure; $\boldsymbol{V}_e$ mean velocity). I will show by a simple example the different points of view involved in using these equations. Consider a box (Fig. 1) containing a gas of free electrons in a crossed electric and magnetic field. If we neglect encounters between the electrons, we know the solution of eq. (1): every electron moves with the same mean velocity $c\,E/H$ perpendicular to both fields, while, according to eq. (2), a stationary state without any mean motion exists if the density of electrons is distributed according to

$$\operatorname{grad} p_e = -e n_e \boldsymbol{E}, \tag{3}$$

just as if there were no magnetic field. The solution of this paradox has been found by T. G. Cowling [1]: At a particular fixed point P inside the box there is a greater chance to observe an electron in the upper part of its trochoidal path (in the orientation

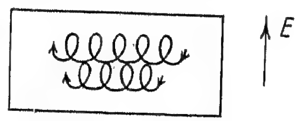

Fig. 1.
The paths of two electrons in crossed electric and magnetic fields. Electric field as shown; magnetic field perpendicular to plane of drawing, pointing to reader.

higher than above, if the density is distributed according to (3). Now the velocity in the upper parts of a trochoid is opposite to the mean motion, and so the average velocity in a fixed point is zero, if the validity of eq. (3) is assumed. That the average velocity of each single electron is also zero comes about by the reflection on the side walls and on the bottom wall of the box. (For the discussion of two other similar paradoxa see A. Schlüter [2]).

To get now the basic equations for the discussion of the conductivity we write an equation for the ions (which we suppose to be singly charged) corresponding to eq. (2) and add to both equations a term describing the friction between electrons and ions. We neglect the internal friction of the electrons (respectively, the ions) with themselves, because this will not influence the electric current in first approximation. So we arrive at the following set of equations:

$$\left.\begin{aligned} \varrho_e \frac{\mathrm{d}\boldsymbol{V}_e}{\mathrm{d}t} &= -en_e\left(\boldsymbol{E} + \frac{\boldsymbol{V}_e \times \boldsymbol{H}}{c}\right) - \operatorname{grad} p_e + n_e \frac{m_i m_e}{m_i + m_e}\,\nu(\boldsymbol{V}_i - \boldsymbol{V}_e) \\ \varrho_i \frac{\mathrm{d}\boldsymbol{V}_i}{\mathrm{d}t} &= en_i\left(\boldsymbol{E} + \frac{\boldsymbol{V}_i \times \boldsymbol{H}}{c}\right) - \operatorname{grad} p_i + n_i \frac{m_i m_e}{m_i + m_e}\,\nu(\boldsymbol{V}_e - \boldsymbol{V}_i) \end{aligned}\right\} \quad (4)$$

(ν number of effective encounters between electrons and ions per particle and per sec).

The friction term corresponds to the first approximation in the kinetic theory of gases [3]. Obviously the number of encounters does not depend markedly on the magnetic field strength in a plasma of given constitution and temperature.

To cast eq. (4) into a more useful form we introduce (supposing $n_i \approx n_e = n$) the velocity of mass motion

$$\boldsymbol{V} = (m_i \boldsymbol{V}_i + m_e \boldsymbol{V}_e)/(m_i + m_e)$$

and the density of electric current

$$\boldsymbol{j} = en(\boldsymbol{V}_i - \boldsymbol{V}_e).$$

Supposing for simplicity $\mathrm{d}\boldsymbol{V}/\mathrm{d}t = \mathrm{d}\boldsymbol{j}/\mathrm{d}t = 0$ we arrive at the equations

$$\left.\begin{aligned} \boldsymbol{j}/\sigma &= \boldsymbol{E}^* - (\boldsymbol{j} \times \boldsymbol{H})\frac{(m_i - m_e)}{ec\varrho} + \boldsymbol{E}' \\ \operatorname{grad} p &= (\boldsymbol{j} \times \boldsymbol{H})/c \\ \boldsymbol{E}' &= (m_i \operatorname{grad} p_e - m_e \operatorname{grad} p_i)/(e\varrho) \\ (\varrho = \varrho_i + \varrho_e,\quad p &= p_i + p_e,\quad \boldsymbol{E}^* = \boldsymbol{E} + (\boldsymbol{V} \times \boldsymbol{H})/c). \end{aligned}\right| \quad (5)$$

Here $\sigma = e^2\varrho/m_i m_e \nu$ sec^{-1} represents the electrical conductivity (in electrostatic units) as found without the presence of a magnetic field. If one neglects the term $\boldsymbol{E}'$ as a "pressure-correction" to the electric current, the first of the eqs. (5) seems to indicate the intuitively expected reduction of the effective conductivity by virtue of the term $\boldsymbol{j} \times \boldsymbol{H}$. But this conclusion is contestable since all equations have to be solved simultaneously and the pressure is therefore determined by the current and the magnetic field. This is made more clear by using a set of equations which is strictly equivalent to (5):

$$\left.\begin{aligned} \boldsymbol{j}/\sigma &= \boldsymbol{E}^c + \boldsymbol{E}'' \\ \operatorname{grad} p &= (\boldsymbol{j} \times \boldsymbol{H})/c \\ \boldsymbol{E}'' &= (m_e \operatorname{grad} p_e - m_i \operatorname{grad} p_i)/e. \end{aligned}\right\} \qquad (6)$$

(Note: if $p_e = p_i$ then $\boldsymbol{E}' = -\boldsymbol{E}''$).

This set is obviously simpler than the previous one, but of course all equations still have to be solved simultaneously. In most cases the only interesting component of the electric field is the one that is parallel to the current, since the other components are often produced by space charges only, and, in any case, they do no work on the current. Now in the simplest cases $\boldsymbol{E}''$ (as $\boldsymbol{E}'$ in (5)) will be perpendicular to $\boldsymbol{j}$ and then it follows directly from (6) that the ratio of $\boldsymbol{j}$ to the component of $\boldsymbol{E}^c$ (that is the electric field as measured by an observer moving with the mass velocity $\boldsymbol{V}$) in its own direction is simply given by the unreduced value σ of the electrical conductivity.

We may discuss the conductivity also from the point of view of the energy balance. According to Maxwell's theory the following equation holds quite generally for the changes of the energy content of an electromagnetic field:

$$\frac{1}{8\pi}\frac{\partial}{\partial t}(H^2 + E^2) + \frac{c}{4\pi} \operatorname{div} (\boldsymbol{E} \times \boldsymbol{H}) + \boldsymbol{V} \cdot (\boldsymbol{j} \times \boldsymbol{H}) = -\boldsymbol{E}^c \cdot \boldsymbol{j}. \qquad (7)$$

In the cases we are interested in, the first term essentially represents the change of the energy density of the magnetic field (with $E^2 \ll H^2$); the second term expresses the energy transport by the Poynting vector, this is very nearly [since $\boldsymbol{E} \approx -(\boldsymbol{V}/c) \times \boldsymbol{H}$] the convective transport of magnetic energy by the matter; the third term is the work done by the Lorentz force. The right side corresponds to real creation and annihila-

tion of the magnetic lines of force. If we now introduce our eq. (5) or (6) we arrive at

$$\boldsymbol{j} \cdot \boldsymbol{E}^r = j^2/\sigma - (m_i/\varrho e)\,\boldsymbol{j} \cdot \operatorname{grad} p_i + (m_e/\varrho e)\,\boldsymbol{j} \cdot \operatorname{grad} p_e. \tag{8}$$

This equation is correct even in non-stationary cases. In the case of considerable admixture of neutral particles (H I regions) it becomes somewhat more complicated but all conclusions remain essentially unaltered.

The term j^2/σ is dissipative and clearly refers to the spontaneous decay of the magnetic field by Joule-losses, while the other terms describe the creation or annihilation (as the case may be) of magnetic flux by the partial pressures, that is to say by the "impressed electric field". If there is *no* magnetic field, some of it will be created by these terms; if there is a *small* magnetic field, they may strengthen or weaken it; but if the magnetic field is so *strong* as to determine the dynamical behaviour of the whole gas, grad p_e and grad p_i will be almost certainly parallel to $(\boldsymbol{j} \times \boldsymbol{H})$ and these terms will nearly cancel.

It follows that in the important question of the lifetime of an interstellar magnetic field we have to take the unchanged value of the electrical conductivity.

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CHAPTER 10

DISCUSSION ON ELECTRICAL CONDUCTIVITY IN GAS DYNAMICS

Chairman: Dr. E. C. BULLARD

COWLING: The question with which Dr. Schlüter has been dealing is simply how much reduction in the conductivity ought one to expect because of the presence of the magnetic field in the interstellar gas? The first calculations seemed to indicate that there is a very considerable reduction in the conductivity for a current perpendicular to the magnetic field. Schlüter has shown that this conclusion is not necessarily valid, since extra currents, that arise in consequence of the presence of pressure fields, eventually restore the conductivity to the ordinary value that holds in the absence of the magnetic field.

This may be put a little more clearly by considering the two ways in which the conductivity may enter into the equations: (a) in the expression j^2/σ for the Joule heat, and (b) in Ohm's law $\boldsymbol{j} = \sigma \boldsymbol{E}$. In generalising one's results to an ionized gas in a magnetic field, j^2/σ still gives the Joule heat with σ unaffected by the magnetic field; for the heating occurs by the conversion of current motion into random motion at collisions, and the magnetic field does not increase the number of collisions. Thus the decay of interstellar magnetic fields is governed by the unreduced conductivity. In generalizing Ohm's law it does not matter what conductivity is used, provided the equations are written completely. Schlüter derived an equation using the unreduced value, but an equivalent equation involving a reduced conductivity can also be used. Both equations involve, in addition to the true electric force $\boldsymbol{E}$, an equivalent electric force, due to pressure gradients; the essential is that, whichever form of equation is used, this equivalent electric force can in no wise be neglected. I cannot say that one of the equations has the more adequate form.

BONDI: It appears to me that in Dr. Schlüter's treatment the pressures are considered as given. However, if th[illegible]

up by transport of electrons and protons, must one not rather consider the pressures as being set up by the field itself?

Further I should like to ask about the definition of the current $\boldsymbol{j}$.

COWLING: In discussing cosmical magnetic fields, the whole normal approach to a problem must be inverted. In discussing a laboratory problem, one asks what electric force is available; the current is given by $\boldsymbol{j} = \sigma \boldsymbol{E}$ and the magnetic force is then found from curl $\boldsymbol{H} = 4\pi\boldsymbol{j}$. But in a cosmic field self-induction is strong and the conductivity has little influence on the magnetic field. The equation curl $\boldsymbol{H} = 4\pi\boldsymbol{j}$ determines what current is required to produce the given $\boldsymbol{H}$, and the equation $\boldsymbol{j} = \sigma \boldsymbol{E}$ simply determines what electric force and in consequence what motion across the lines of force, is required to produce the currents.

SCHLÜTER: If an appreciable magnetic field exists, this field and the pressures cannot be considered as independent. The equations which I deduced have then to be solved as a set of simultaneous equations.

LIEPMANN: There seems to be a difference with ordinary hydrodynamics and turbulence in so far as the electric forces are long range forces. This can influence the definition of mean values and the application of an ergodic property.

SCHLÜTER: The mean values should be determined over distances which are large compared with the radius of curvature of the trajectories of the electrons. The range of the Coulomb forces is effectively limited by polarization of the plasma, analogous to the Debye-Hückel ionic cloud in electrolytes.

The friction term may be assumed to be linear in the relative velocity as long as this velocity is small compared with the thermal velocities.

BIERMANN: When speaking of the definitions of electrical conductivity, it is useful to look at the history of the subject. The tensorial form of the expression for the conductivity has been introduced for the first time in the theory of the ionosphere, where the use of a measure of the conductivity depending on the local magnetic field cannot be avoided. A treatment of this form has also been used in the work done in Göttingen in this field. Lucas and Schlüter (see Naturw. **40**, 239, 1953; Archiv für elektr. Uebertragung, **8**, 27, 1954). The next problem was the dissipation of magnetic energy by Joule losses in the solar corona and in interstellar space. Here the use of the form appropriate for the ionosphere has resulted in serious misunderstandings. We considered it in relation with our work on the origin of magnetic fields in ionized gaseous

masses by impressed electromotive forces, and it was found that for these problems a scalar expression for the conductivity not depending on the magnetic field strength, is both simpler and more appropriate. This has been a main topic of Dr. Schlüter's contribution. Of course it is true that every definition when properly applied gives the right result; but experience has shown that the amount of theoretical effort involved may be rather different, and when a new theoretical problem is approached it is very important to have at hand a formalism appropriate to the problem in question.

HOYLE: There is a rather impressive coincidence between the energy densities of the turbulent motions, of the magnetic fields, and of the cosmic rays. For that reason it is possible that the cosmic rays might be produced by electromagnetic processes in interstellar space. The problem is whether a motion once started can lead to the appearance of fields which accelerate the motion, as occurs in a betatron. If the answer is yes, then Bondi's suggestion that the betatron effect is a more powerful source of dissipation than conductivity may be right.

SCHLÜTER: The magnetic and kinetic energies are coupled by the equation of motion. The energy density of the magnetic field now gives an upper limit to the energy density of the cosmic radiation, if in *intergalactic* space there is neither a magnetic field nor cosmic radiation of comparable intensity.

We have also investigated the betatron effect and found that it requires very unusual conditions, namely a very small radius of curvature of the lines of the electric current maintaining the magnetic field and a very small density (see: A. Schlüter, Zeitschrift f. Naturforschung **7a**, 136, 1952).

BULLARD: Is the radiation from the electrons moving in their circular paths always negligible?

SCHLÜTER: Radiation by individual electrons is neglected, since a hydrodynamic description of the electron gas has been used. The time of decay of the kinetic energy of single non-relativistic electrons due to radiation in a magnetic field is $2{,}6.10^8\, H^{-2}$ sec (see: H. Alfvén, Cosmical Electrodynamics, p. 35). It is only appreciable for electrons of extreme relativistic energy.

TEMPLE: What about the relativistic corrections?

SCHLÜTER: They are not needed if the velocities are $< c$ and the times involved are longer than the time that light needs to cross the system.

DEUTSCH: So far you have treated a fully ionized gas. What changes are needed in a partially ionized gas (e.g. a H I region)?

SCHLÜTER: The influence of the neutral particles on the conductivity in H I-regions is negligible. For the motion of the entire gas the neutral particles are important as their diffusion relative to the charged particles is small and for this question the Lorentz force may be thought of as acting on the whole gas. Of course this "ambipolar diffusion" represents an additional sink of energy.

SCHATZMAN: Are plasma oscillations included in your formulae?

SCHLÜTER: They can be derived if the inertia terms are retained. That is easily done. Also Alfvén's waves may be found (see: A. Schlüter, Ann. d. Physik **10**, 418, 1952).

CHAPTER 11

THE ENERGY BALANCE OF THE INTERSTELLAR MEDIUM

BY

M. P. SAVEDOFF

*Temporarily at Leiden**

In the investigation of the gas dynamics of the interstellar medium, information is needed on the rate of change of the energy density to make possible discussion of processes in which the departure from adiabatic conditions is important. Equilibrium temperatures were obtained by Spitzer and his collaborators by determining at what temperature the rate of change in the energy content vanishes. Unfortunately, the numerical data from which the temperatures were computed were not published at that time, and it has proven difficult to estimate the rate of heating or cooling of the interstellar medium without elaborate computation. It is hoped that the tables included in this note will make possible rapid estimates of which processes are important and of the approximate rate of change of the energy density for all gas densities and temperatures. The physical processes, constants, and notation are all taken from the series of papers entitled *Temperature of Interstellar Matter* [1] and include all modifications consistent with paper III. I wish to emphasize that, although this note is prepared by M. Savedoff, the original computations were the work of W. Buscombe, M. Savedoff, and J. D. Schopp, under the guidance of L. Spitzer.

The following values can be assumed for the abundances of the various constituents of the gas.

RELATIVE ABUNDANCES

	n_H	n_p	n_e	n_i	n_a	n_g	n_m
H I region	1	0	5×10^{-4}	5×10^{-4}	1.5×10^{-3}	10^{-12}	0.1 **
H II region	0	1	2×10^{-3}	2×10^{-3}	0	10^{-12}	0

H: hydrogen atoms
p: protons
e: electrons
i: other ions
a: other atoms
g: grains
m: molecules (H_2)

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** $n_m = 0.1$ if $n_g > 10^{-12}$ and $T < 5{,}000°$; otherwise $n_m = 0$.

I. ENERGY PROCESSES IN H II REGIONS

In H II regions, the principal source of energy is superelastic collisions of electrons with protons, ions, and grains. Here a free electron encounters a charged particle and is captured to a bound state emitting a photon and subsequently emitting more photons until it is in the ground state. On absorbing another photon from the galactic radiation field, the free electron is returned, but with a kinetic energy approximately equal to the difference in energy of the photon absorbed and the photon emitted. Thus in the Tables, G_{ep} is the product of the mean energy gained per electron capture and the rate of capture per electron. The photon density is only important at much higher densities. G_{ei} and G_{eg} are comparable to G_{ep}, but refer to encounters with ions and grains respectively.

Encounters of electrons with cosmic rays are probably important only at densities of less than 10^{-5} hydrogen atoms per cubic centimeter. Since protons far outnumber all other constituents, only the term G_{ep} need be included among the gains, the next contributor being $G_{ei} \approx (n_i/n_p)\, G_{ep}$.

The G terms represent processes which increase the mean kinetic energy of the electrons. These terms are balanced by processes which convert the kinetic energy into radiation. For instance, the loss L_{ep} represents the energy loss per electron by free-free transitions. Some of this energy is observable as radio frequency radiation. In collisions of electrons with atoms or ions having bound excited states, these states may be excited. Subsequently, the return to the ground state may take place either through radiational or collisional de-excitation. L_{ei} is the energy thus radiated per electron per second. At higher densities collisional de-excitation becomes important. The electron density at which L_{ei} is reduced a factor 2 by de-excitation is also included. Unless otherwise stated, the Tables which follow refer to the standard case of paper III, with $n_p = 1$.

In Table II, the rate of change of the energy density is given for a region near an O-star with a density of one proton per cubic centimeter and for different values of the cross section ion density product.

For other densities, the numbers of Table II can be multiplied by $n_p{}^2$ to give the rate of change of the energy density, provided that the effects of electron saturation are not important at this density. (Note that it is most dangerous to interpolate for intermediate temperatures in L_{ei}, as there are several processes operative simultaneously).

TABLE I

H II Region

Energy gain (or loss) in ergs/second/electron, assuming $n_p = 1$
(linear density dependence below saturation)

T	G_{ep}			
	Galaxy	O-Star	B-Star	A-Star
10	$+1.39 \times 10^{-22}$	$+2.12 \times 10^{-22}$	$+9.31 \times 10^{-23}$	$+4.40 \times 10^{-23}$
10^2	$+3.50 \times 10^{-23}$	$+4.35 \times 10^{-23}$	$+2.34 \times 10^{-23}$	$+1.10 \times 10^{-23}$
10^3	$+8.01 \times 10^{-24}$	$+1.02 \times 10^{-23}$	$+5.28 \times 10^{-24}$	$+2.37 \times 10^{-24}$
10^4	$+1.23 \times 10^{-24}$	$+1.65 \times 10^{-24}$	$+6.54 \times 10^{-24}$	$+4.3 \times 10^{-26}$
10^5	-3.37×10^{-25}	-2.68×10^{-25}	-4.31×10^{-25}	-5.32×10^{-25}

continuation

T	G_{ei}	G_{ec}	L_{ep}	L_{ei}	$(n_e)_{sat}$
10	$<2 \times 10^{-25}$	10^{-29}	$+4.49 \times 10^{-27}$	$+2.3 \times 10^{-31}$	10^2
10^2	$<7 \times 10^{-26}$	,,	$+1.42 \times 10^{-26}$	$+4.1 \times 10^{-25}$	$10^{2.3}$
10^3	$<1.6 \times 10^{-26}$	,,	$+4.49 \times 10^{-26}$	$+9.2 \times 10^{-25}$	10^3
10^4	$<2 \times 10^{-27}$	,,	$+1.42 \times 10^{-25}$	$+8.1 \times 10^{-24}$	$10^{5.5}$
10^5	$< 10^{-28}$	,,	$+4.49 \times 10^{-25}$	$+6.0 \times 10^{-23}$	$10^{6.3}$

TABLE II

H II Region

Radiation rate in ergs/second/electron for region $n_p = 1$ near O-star
(linear density dependence below saturation)

Effective n_i	T					$T_{equ.}$
	10	10^2	10^3	10^4	10^5	
Standard	-2×10^{-22}	-4.30×10^{-23}	-9.8×10^{-24}	$+6.5 \times10^{-24}$	$+6.1\times10^{-23}$	6,900
10^{-1} Standard	-2×10^{-22}	-4.35×10^{-23}	-1.01×10^{-23}	-7.0×10^{-24}	$+6.8\times10^{-24}$	12,000
10^{-2} Standard	-2×10^{-22}	-4.35×10^{-23}	-1.02×10^{-23}	-1.43×10^{-24}	$+1.3\times10^{-24}$	24,000

II. ENERGY PROCESSES IN H I REGIONS

Electron capture and subsequent radiative ionization is the principal method of transfer of energy from the radiation field to the kinetic energy of the particles in H I regions. G_{eH} and G_{ei} represent the energy production per electron per second with H (to form H^-) and the ions C II, Fe II, Mg II, Al II, S II, etc., respectively. Both nitrogen and oxygen are neutral in these regions (because of their high ionization potential) and do not contribute to the energy production. G_{ec} and G_{Hc}

are 10^{-29} and 8.2×10^{-27} respectively at all temperatures and densities (gains from cosmic rays).

TABLE III

H I Region

Energy gain (or loss) in ergs/second/electron, for region $n_H = 1$ (linear density dependence below saturation)

T	G_{eH}	G_{ei}	L_{eH}	$(n_H/n_e)L_{Hg}$	$(n_H/n_e)L_{Hm}$
10^1	$+1.24\times10^{-29}$	$+3.95\times10^{-26}$	$+2.8\times10^{-33}$	-6.0×10^{-29}	$+1.4\times10^{-47}$
10^2	$+1.22\times10^{-28}$	$+9.05\times10^{-27}$	$+7.7\times10^{-31}$	$+1.5\times10^{-27}$	$+2.8\times10^{-25}$
10^3	$+1.08\times10^{-27}$	$+1.80\times10^{-27}$	$+1.1\times10^{-28}$	$+5.8\times10^{-26}$	$+2.4\times10^{-22}$
10^4	-2.36×10^{-27}	$+2.00\times10^{-28}$	$+6.7\times10^{-27}$	$+1.9\times10^{-24}$	Dissociation
10^5	$-6.5\ \times10^{-26}$	-3.55×10^{-29}	—	$+6.0\times10^{-23}$	Dissociation

(continuation)

T	L_{ei}	(n_e)sat	L_{ea}	(n_e)sat	L_{em}	(n_e)sat
10^1	$+5.5\times10^{-32}$	10^2	$+6.8\times10^{-36}$	$10^{5.7}$	$+10^{-287}$	$10^{3.2}$
10^2	$+1.1\times10^{-25}$	$10^{2.3}$	$+1.2\times10^{-28}$	$10^{5.2}$	$+10^{-40}$	$10^{2.7}$
10^3	$+2.3\times10^{-25}$	$10^{3.3}$	$+2.1\times10^{-27}$	$10^{4.1}$	$+3.8\times10^{-24}$	$10^{2.2}$
10^4	$+1.9\times10^{-24}$	$10^{5.6}$	$+2.3\times10^{-25}$	10^7	Dissociation	
10^5	$+1.5\times10^{-23}$	10^7	$+8.6\times10^{-26}$	$10^{6.4}$	Dissociation	

$G_{ec} = 10^{-29}$; $(n_H/n_e)G_{Hc} = 8.2\times10^{-27}$.

Appreciable losses are produced through collisional excitation of bound states in ions, atoms, and molecules (H_2) which are represented by L_{ei}, L_{ea}, L_{em}, and L_{Hm}. For H_2, excitation of vibrational levels by electron collisions (L_{em}) has been separated from excitation of rotational levels by atomic hydrogen collisions (L_{Hm}).

Another source of energy loss is in the radiative cooling of the grains. The last discussion by van de Hulst indicates the reliability of the numerical data for this process. The numerical data in Table III for L_{Hg} are calculated with a particle of 0.1 μ radius, temperature of the grain (T_g) of 20° K, and a density of 10^{-12} grains per H atom.

The photo-emission of electrons by the grains could be an important source of energy. For dielectric grains this effect is negligible. G_{eg} for a moderately photo-emissive surface is given in Table IIIa for different densities. Extrapolated values are indicated as uncertain.

TABLE IIIa

H I Region

G_{eg} in ergs/second/electron for a moderately photo-emissive grain

T	n_H (cm^{-3})		
	10^{-2}	10^{0}	10^{2}
10^{1}	$\sim 6 \times 10^{-27}$	$\sim 7 \times 10^{-27}$	10^{-27}
10^{2}	3.9×10^{-27}	7×10^{-27}	1.7×10^{-27}
10^{3}	2.6×10^{-27}	$\sim 4 \times 10^{-27}$	$\sim 4 \times 10^{-27}$
5×10^{3}	1.5×10^{-27}	2.8×10^{-27}	2.7×10^{-27}

F. D. Kahn [2] estimates that, in addition to energy from the radiation field, the systematic motion of clouds is an important source of energy. He obtains $G_{H\,\text{kinetic}} = 3.1 \times 10^{-27}$ (independent of the density), i.e., with $n_e/n_H = 5 \times 10^{-4}$, the value $(n_H/n_e)\, G_{H\,\text{kinetic}} = 6.4 \times 10^{-24}$, that may be compared to the entries of Tables III, IIIa, and IV,

Again the rate of change of energy density is given in Table IV for an H I region with no metallic grains and in which cloud collisions are ignored. The equilibrium temperature is also given. These apply to the standard composition with non-photo-emissive grains.

TABLE IV

H I Region

Radiation rate in ergs/second/electron for standard abundances (dielectric grains)

T	n_H (cm^{3})				
	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
10^{1}	-8.6×10^{-27}	-1.2×10^{-26}	-4.8×10^{-26}	-4.0×10^{-25}	-4.0×10^{-24}
10^{2}	-7.2×10^{-27}	$+2.0 \times 10^{-27}$	$+9.4 \times 10^{-26}$	$+3.8 \times 10^{-24}$	$+3.8 \times 10^{-23}$
10^{3}	6.3×10^{-27}	$+2.1 \times 10^{-26}$	$+2.9 \times 10^{-25}$	$+2.4 \times 10^{-21}$	$+2.4 \times 10^{-20}$
10^{4}	$+3.0 \times 10^{-26}$	$+3.7 \times 10^{-25}$	$+3.8 \times 10^{-24}$	$+3.8 \times 10^{-23}$	$+3.8 \times 10^{-22}$
10^{5}	$+7.5 \times 10^{-25}$	$+7.5 \times 10^{-24}$	$+7.5 \times 10^{-23}$	$+7.5 \times 10^{-22}$	$+7.5 \times 10^{-21}$
T_{equ}	—	—	47.3	—	42.6

REFERENCES

[1] Astrophysical Journal, **107**, 6 (1948); **109**, 337 (1949); **111**, 593 (1950).

[2] F. D. Kahn, Preliminary Communication to the Symposium; see also Ch. 12.

CHAPTER 12

THE HEATING AND COOLING OF INTERSTELLAR GAS CLOUDS

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1. CONDITIONS IN INTERSTELLAR SPACE

The gas in interstellar space consists largely of atomic hydrogen, with a smoothed-out density of 1 atom per cm^3, and is present in discrete clouds. The average temperature of the gas, as observed by the intensity of the 21-cm line of hydrogen, is about 100° K, corresponding to a sonic velocity of about 1 km/sec. The average radial peculiar velocity of the clouds is, however, 5 km/sec, so that the relative motions of the clouds appear to be highly supersonic.

A line of sight 1 kiloparsec ($= 3{,}08.\ 10^{21}$ cm) in length cuts eight to twelve separate clouds on the average, say 10 for the purposes of this paper, and the clouds occupy about 5 per cent of interstellar space. These values were all quoted by Oort in the Henry Norris Russell lecture of 1951 [1].

To complete the data, the following composition is adopted for the more important elements in the gas. It is taken from a table compiled by Aller [2]

TABLE I

Abundance of the Elements

Element	Const. + $\log_{10} n$	Const. + $\log_{10} \varrho$	ϱ/ϱ_H
H	13.27	13.27	1
He	12 41	10.0	0.54
Ne	10..04	11.30	0.01
O	10.0	11.20	0.01
N	9.49	10.64	0.004
C	9.23	10.31	0.001
Mg	9.05	10.44	0.001
Ar	9.0	10.6	0.002
Si	8.80	10.08	0.0006
S	8.52	10.02	0.0006
Fe	8.48	10.23	0.001

In this table:

n = abundance, by number of atoms per cm^3,
ϱ = abundance, by mass, of element per cm^3,
ϱ_H = abundance, by mass, of hydrogen per cm^3.

There is some uncertainty about the values quoted for the inert gases.

A highly idealised model can now be constructed for the interstellar gas. Let all clouds be spheres of equal radius and density. Then

$$(4\,\pi/3)\,Na^3 = r \tag{1}$$

$$\pi\,Na^2 l = s, \tag{2}$$

where N = number of clouds per unit volume, a = radius of a cloud. Relation (1) states that a fraction r of space is occupied by clouds, and relation (2) that a line of sight of length l cuts s separate clouds on the average. With the present data

$$r = 0.05, \quad s = 10, \text{ when } \quad l = 1 \text{ kpc} = 3 \times 10^{21} \text{ cm.}$$

By (1) and (2),

$$a = 3\,rl/4\,s, \qquad N = 16\,s^3/9\pi r^2\,l^3,$$

so that $a = 1.1 \times 10^{19}$ cm $= 3.75$ ps; $N = 9 \times 10^{-60}$ cm^{-3},

corresponding to a density of one cloud in every 4,100 ps^3. The average density of matter in a cloud is 20 H atoms/cm^3 or 3.3×10^{-23} gm/cm^3, so that a cloud has a mass 1.8×10^{35} gm, equal to ninety times the mass of the Sun.

The cross-section for a collision between two clouds is $4\,\pi\,a^2$, but in general not all parts of both clouds are involved. A given element of volume will take part in a collision only when it passes within a distance a of the centre of another cloud and the cross-section for such a collision is $\pi\,a^2$, or one quarter as large.

We assume that the velocity components of the clouds have a Maxwellian distribution. Thus $p(v)\mathrm{d}v$ is the probability that the velocity component in a given direction should lie between v and $v + \mathrm{d}v$, where

$$p\,(v) = \frac{1}{v_0\sqrt{\pi}} \cdot e^{-v^2/v_0^2}. \tag{3}$$

The average peculiar radial component is

$$2 \int_0^\infty v\, p(v)\, \mathrm{d}v = \frac{v_0}{\sqrt{\pi}} = 5 \text{ km/sec},$$

with the present data. Thus $v_0 = 9$ km/sec. The number of collisions between clouds, per unit volume and unit time, in which the relative velocity of the cloud centres lies between V and $V + \mathrm{d}V$ is

$$\mathfrak{N}(V)\, \mathrm{d}V = \frac{N^2}{v_0^3 \sqrt{2\pi}}\, 4\, \pi\, a^2 V^3 e^{-V^2/2v_0^2}\, \mathrm{d}V.$$

This formula is adapted to the present case from an equation given by Jeans [3]. The total number of collisions per unit volume and unit time is $4\, \pi\, a^2 N^2 v_0 \sqrt{2/\pi}$. Since two clouds are involved in each collision the frequency of collisions per cloud is $8\pi\, a^2 N v_0 \sqrt{2/\pi}$. With our adopted values this becomes 1.9×10^{-14} collisions per cloud per second, that is a collision once in every 5.2×10^{13} sec or once in 1.7×10^6 years. A particular part of a cloud suffers collisions at one quarter this rate, that is once in 2.1×10^{14} sec, or once in 6.8×10^6 years.

It is of interest to find $\overline{V}$, $\overline{V^2}$, the mean values of the relative velocity and of its square. We find

$$\overline{V} = \tfrac{3}{4}\, v_0 \sqrt{2\pi} = 17.5 \text{ km/sec} = 1.75 \times 10^6 \text{ cm/sec}$$

and

$$\overline{V^2} = 4\, v_0^2 = 324\, (\text{km/sec})^2 = 3.24 \times 10^{12}\, (\text{cm/sec})^2.$$

A detailed investigation into the dynamics of a collision would be very difficult. But perhaps some suggestions may be made about its probable results.

A collision will be completely inelastic, and will generate considerable heat in the clouds that take part in it. After a collision one, two or three separate clouds may be formed. The following are three typical cases.

(i) Two clouds of equal mass and size collide; before the impact the relative velocity V lies along the line of centres. Then the two clouds merge into one. Each cloud loses kinetic energy corresponding to a speed $\frac{1}{2} V$ and there is a gain of thermal energy of $1/8\ V^2$ per unit mass.

(ii) Two clouds of equal mass and size collide; before collision the relative velocity does not lie along the line of centres. Then three new clouds are formed. Two of these are the parts of each cloud which did

not overlap the other cloud, and they travel on as if nothing had happened. The third cloud consists of the overlapping parts and it is heated, as in (i).

(iii) Several possibilities exist when clouds of unequal size collide. Thus a small cloud may be swallowed up by a large one, but if the small cloud has a much higher density it may burrow its way through the large cloud and gain mass.

In these ways clouds can merge, split up and gain or lose mass. The continued occurrence of inelastic collisions does not necessarily lead to the formation of a large single cloud.

The average gain of thermal energy during collisions is easily estimated. With the present data it is $1/8\ \overline{V^2} = 4 \times 10^{11}$ erg per gm per collision, i.e. once in 2.1×10^{14} sec and corresponds to a temperature increase of 3,100° K. The average rate of heating by collisions is 3.1×10^{-27} erg per H atom per sec, or 6.2×10^{-26} erg per cm^3 per sec in a cloud which contains 20 H atoms per cm^3.

The clouds can regain kinetic energy by falling towards the galactic centre. Thus in falling from infinity to a distance R from the centre a cloud has lost potential energy GM/R per unit mass, where M is the mass of the galaxy, supposed largely concentrated at the centre. A cloud moving in a circular orbit of radius R has a speed U given by $U^2 = GM/R$ and has therefore a kinetic energy $\frac{1}{2}U^2 = \frac{1}{2}\ GM/R$ per unit mass. In falling from infinity such a cloud must have dissipated an amount $\frac{1}{2}\ U^2$ of energy per unit mass. Oort estimates that in the neighbourhood of the Sun $U = 260$ km/sec $= 2.6 \times 10^7$ cm/sec, and so the energy dissipated is 3.4×10^{14} erg per gm, or sufficient for 850 collisions. With one collision in 6.8×10^6 years, this is enough for 6×10^9 years.*

2. THE HEATING AND COOLING OF THE CLOUDS

The rate at which the collisions heat the clouds is far greater than that due to any other plausible mechanism. It is usually assumed that the H I regions of space are heated mainly by the photo-ionization of atoms [4]. It can be shown that carbon will be the most important ionized element, that it will be present almost entirely in the form of C^+ ions [5] and thus the rate of ionization of C atoms is determined by the rate of recombination of the ions with electrons. If the ionizing radiation comes from a star at a temperature $T^{\star}$ (usually taken to be 10,000° K) and the

* Many ideas in this section arose out of a very interesting discussion with Dr. V. Vand.

gas in the cloud is much cooler than this, then the average energy absorbed in one ionization is $kT^\star$ on the average.

We estimate the number of ionizations and recombinations per unit volume and unit time. The density n_e of electrons is approximately equal to n_i, the density of C^+ ions, since carbon is the most abundant element which can be ionized. The rate of recombination is then $\alpha\, n_i^2$ processes per cm³ per sec. Here α is the recombination coefficient, which is about 10^{-11} cm³ sec^{-1} at 100° K [5]. The abundance of carbon by number of atoms is about 10^{-4} times that of hydrogen, by Table I, so that $n_i = 2 \times 10^{-3}$ ions per cm³. Hence $\alpha\, n_i^2\, kT^\star = 5.5 \times 10^{-29}$ erg per cm³ per sec.

A similar calculation shows that the photo-ionization of H^- ions may contribute about the same amount of energy. It is clear that the collisions between the clouds provide far more heating than does photo-ionization. (This is no longer true when the H atoms themselves are ionized.)

Now the clouds are raised by the collisions to 3,100° K once in every 2.1×10^{14} sec, on the average. The observations of the 21-cm line of hydrogen yield a temperature of 100° K, and there must be an efficient process which cools the gas between collisions. It seems that hydrogen molecules are the most effective cooling agents. According to Spitzer's results [6] they radiate energy quite rapidly at high temperatures but become progressively less efficient at lower temperatures (say below 500° K).

The rate of change of energy per unit volume at a temperature T, below 500° K is

$$n_{\mathrm{H}} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{3}{2}\, kT\right) = -\gamma\, n_H\, n_{\mathrm{H}_2},$$

where n_{H} = no. of H atoms per cm³

n_{H_2} = no. of H_2 molecules per cm³

$\gamma = 2.5 \times 10^{-26} \sqrt{T}\, e^{-520/T}$ (this is the value found theoretically by Spitzer, but written in a different form).

Then

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{2\gamma}{3k}\, n_{\mathrm{H}_2} = -1.2 \times 10^{-10}\, n_{\mathrm{H}_2} \sqrt{T}\, e^{-520/T}. \tag{4}$$

This gives the variation of temperature with time. We can now estimate n_{H_2} by finding out how many hydrogen molecules are required

per unit volume in order to cool the gas sufficiently to explain the results of the 21-cm measurements.

The cooling from 3000° K to about 500° K takes place quite rapidly but at lower temperatures the factor $e^{-520/T}$ reduces the rate. The gas will, in fact, spend most of its time at a temperature much below 520° K. We shall therefore solve the following problem.

The gas is heated to a temperature of 520° K once in 2.1×10^{14} sec and then allowed to cool. What density of hydrogen molecules is required so that 100° K is the harmonic mean value of the temperature, averaged over the interval between two successive heatings? (It is shown in the appendix that the 21-cm measurements do, in fact, determine the harmonic mean temperature.)

Equation (4) can be re-written in a more convenient form if we put

$$\frac{520}{T} = u^2; \quad 2.63 \times 10^{-12}\, n_{H_2}\, t = \tau,$$

and it becomes

$$\frac{e^{u^2}}{u^2}\frac{du}{d\tau} = 1. \tag{5}$$

When $t = 0$, $\tau = 0$ and $T = 520$, giving $u = 1$.

The harmonic mean of T over the period from time $t = 0$ until $t = t_1$ is

$$T_M = \left[\frac{\int_0^{t_1} \frac{dt}{T}}{t_1}\right]^{-1} = \frac{\tau_1}{\int_0^{\tau_1} \frac{u^2}{520}\, d\tau}.$$

In the present case $T_M = 100°$ K, approximately, so that

$$\frac{1}{\tau_1}\int_0^{\tau_1} u^2\, d\tau = 5.2. \tag{6}$$

By equation (5)

$$\tau_1 = \int_1^{u_1} \frac{e^{v^2}}{v^2}\, dv = -\left[\frac{e^{u_1^2}}{u_1} - e\right] + 2\int_1^{u_1} e^{v^2}\, dv,$$

and

$$\int_0^{\tau_1} u^2\, d\tau = \int_1^{u_1} e^{v^2}\, dv, \quad \text{where } u = u_1 \quad \text{when } \tau = \tau_1.$$

Equation (6) can then be written

$$\int_1^{u_1} e^{v^2}\,dv = -5.2\left[\frac{e^{u_1^2}}{u_1} - e\right] + 10.4\int_1^{u_1} e^{v^2}\,dv$$

or

$$\frac{5.2\,e^{u_1^2}}{u_1} = 9.4\int_0^{u_1} e^{v^2}\,dv + 5.2\,e - 9.4\int_0^1 e^{v^2}\,dv$$

or

$$\frac{1}{u_1} = 1.81\,e^{-u_1^2}\int_0^{u_1} e^{v^2}\,dv + e^{-(u_1^2-1)}\left[1 - 1.81\,e^{-1}\int_0^1 e^{v^2}\,dv\right]. \quad (7)$$

A table of the function $e^{-u^2}\int_0^u e^{v^2}\,dv$ is reproduced in Unsöld's "Physik der Sternatmosphären"[7]. According to the values given there the term in square brackets in equation (7) equals 0.02. As $e^{-(u_1^2-1)}$ is also small the second term on the right hand side can be neglected.

From the values in the table the solution of the equation

$$\frac{1}{u_1} = 1.81\,e^{-u_1^2}\int_0^{u_1} e^{v^2}\,dv$$

can be found to be

$$u_1 = 2.63, \quad \text{so that}$$

$$\tau_1 = 2\int_1^{u_1} e^{v^2}dv - \left(\frac{e^{u_1^2}}{u_1} - e\right) = 40$$

or

$$2.63 \times 10^{-12}\, n_{H_2}\, t_1 = 40. \quad (8)$$

But t_1 = time interval between successive heatings $= 2.1 \times 10^{14}$ sec, so that

$$n_{H_2} = 7.5 \times 10^{-2} \text{ molecules per cm}^3.$$

Since $n_H = 20$ atoms per cm³ in an average cloud, it follows that only 0.4% of the particles present need to be hydrogen molecules, or that only 0.8% of the hydrogen is present in molecular form.

If $T_M = 50°$ K the corresponding value of n_{H_2} must be 34 molecules per cm³, which is too high a density to be possible in an average cloud.

The value of n_{H_2} is very sensitive to variations in T_M. At present the determination of temperatures by 21-cm radiation is not sufficiently accurate to do more than to find the order of magnitude of the abundance of hydrogen molecules.

APPENDIX — THE INTERPRETATION OF THE TEMPERATURE OF INTERSTELLAR CLOUDS AS MEASURED BY MEANS OF THE 21-CM LINE.

It can be shown that the temperature found experimentally is effectively $[\overline{T^{-1}}]^{-1}$, the average being taken over the H atoms along the line of sight.

Consider a hydrogen gas with a non-uniform density n atoms per cm^3 and a non-uniform temperature T. The H atoms have states 1 and 2 separated by an energy difference $h\nu$ ($= 9.7 \times 10^{-18}$ erg in the present case. This is much less than the lowest value of kT in the gas).

Let A = probability of a spontaneous transition $2 \longrightarrow 1$,
B = probability of absorption and of induced emission.

If the population of the two states is determined essentially by collisions between the atoms, we have[8]

$$n_1 = \frac{n}{1 + e^{-h\nu/kT}} \sim \frac{n}{2}\left(1 + \frac{h\nu}{2kT}\right) = \text{density of atoms in state 1,}$$

$$n_2 \sim \frac{n}{2}\left(1 - \frac{h\nu}{2kT}\right) = \text{density of atoms in state 2.}$$

The absorption and emission combine to give an effective absorption coefficient $\frac{B}{c}(n_1 - n_2)\ \text{cm}^{-1} = \frac{Bn\,h\nu}{2c\,kT}\ \text{cm}^{-1} = n\varkappa$, say. Now if an element of length ds at optical depth τ subtends a solid angle dΩ at the Earth, the intensity of radiation received from it is

$$\frac{\mathrm{d}\Omega}{4\pi}\, n_2\, A\, h\nu\, e^{-\tau}\, \mathrm{d}s \sim \frac{\mathrm{d}\Omega}{8\pi}\, n\, A\, h\nu\, e^{-\tau}\, \mathrm{d}s,$$

since $n_2 \sim \frac{1}{2} n$ and in which $\mathrm{d}\tau = n\varkappa\, \mathrm{d}s$.

Now let $\mathrm{d}N = n\mathrm{d}s$, so that N is the number of H atoms per unit column of length s along the line of sight. The intensity of radiation at the Earth becomes

$$\frac{\mathrm{d}\Omega}{8\pi} A\, h\nu \int_0^\infty e^{-\tau}\, \mathrm{d}N, \quad \text{where} \quad \tau = \int_0^N \frac{\sigma}{T}\, \mathrm{d}u$$

and $\sigma = \frac{B\,h\nu}{2\,c\,k}$. We define $T_R = \sigma \int_0^\infty \exp\left[-\int_0^N \frac{\sigma}{T}\,\mathrm{d}u\right] \mathrm{d}N$. This is the temperature that one infers from the results of measurements at 21 cm.

If the temperature is uniform then $T = T_R$ throughout the medium and the radiation received is

$$\frac{d\Omega}{4\pi} \cdot \frac{A\,h\nu}{2\,\sigma} = \frac{d\Omega}{4\pi} \cdot \frac{A}{B}\, c\, kT_R = \frac{c\,d\Omega}{4\pi} \cdot \frac{8\pi\nu^2}{c^3}\, kT_R,$$

by the usual relations between A and B.

The interpretation of T_R is more difficult when T is not uniform. If the smallest regions within which T varies significantly are large enough, the temperature observed at the Earth will be determined solely by the conditions in the nearest clouds. This does not seem to be the case. Then suppose that the fluctuations of T are so rapid that an average of a function of T taken over N becomes approximately equal to the average, taken over N infinite, for $N < N_1$, where N_1 is the number of H atoms in a unit column of unit optical depth. Then the value of T_R does not depend on the nearest clouds.

In this case

$$\frac{1}{N}\int_0^N \frac{1}{T}\, dN = \frac{1}{T_M} + \varepsilon\,(N)$$

and $\varepsilon\,(N)$ is small for $N > N_0$, where $N_0 < N_1$. Thus

$$\tau = \int_0^\infty \frac{\sigma}{T}\, dN = \frac{\sigma N}{T_M} + \sigma\, N\, \varepsilon\,(N),$$

and finally $\displaystyle\int_0^\infty e^{-\tau}\, dN = \int_0^\infty \exp\left\{-\frac{\sigma N}{T_M} - \sigma\, N\, \varepsilon\,(N)\right\} dN \sim T_M/\sigma.$

Thus if T fluctuates rapidly enough, T_R, the temperature inferred from radio measurements, may be identified with T_M, where

$$\frac{1}{T_M} = \lim_{N \to \infty} \frac{1}{N}\int_0^N \frac{1}{T}\, dN.$$

In the evaluation of $\frac{1}{T_M}$ the average is taken over all atoms in the line of sight at a given time. If all atoms are subject to similar physical conditions the same value can be found by averaging over all times for a given set of atoms.

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[8] Added in proof: Inadvertently the statistical weights were taken equal, while actually they are 3 and 1. This does not change the result.

CHAPTER 13

ON THE FORMATION OF CONDENSATIONS IN A GASEOUS NEBULA

BY

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Since a complete paper on this subject will appear in the "Journal of Atmospheric and Terrestrial Physics", [1] we shall restrict ourselves to a brief summary.

The object of the paper is to provide a possible mechanism for explaining the occurrence of condensations in a nebula, exposed either to the highly diluted radiation of a star of very high temperature, or to similar ultraviolet radiation from a different origin, or to high energy corpuscular radiation. In order to bring out the principles as clearly as possible, the situation is simplified to the extreme.

One considers the nebula to consist of a single species of atoms, say nitrogen, which can occur in two stages of ionization. The atoms (or ions) in the lowest stage are called "atoms" A; it is supposed that they have a low metastable level B, excitable by electron impact, the excitation energy being χ_{AB}. For instance, stage A may be N^+, where the spectrum [N II] is emitted by electron impact. The lowest stage A can be ionized to the next stage A^+ (in the preceding example this would be N^{++}), which has no metastable low level. The photo-electrons freed by ionization in the case where this is due to ultraviolet radiation, will have an average kinetic energy $\frac{3}{2} k\bar{T}$, where $\bar{T}$ is called the liberation temperature. This liberation temperature is dependent on the nature of the ultraviolet radiation. When the radiation is coming from a star with black body temperature T_*, $\bar{T}$ will be a function of T_*. In general:

$$\bar{T} < \frac{2}{3} T_*,$$

the sign of equality holding when $k\bar{T}$ is small compared with the ionization energy. The electrons present have an average kinetic energy determined by the ordinary electron temperature T_e, while the average

energy of the electrons when captured by the atoms in the A^+ stage (which then return to the A stage), amounts to $\frac{3}{2}\, k\, \overline{T}_e$, where $\overline{T}_e$ is called the electron capture temperature. In general:

$$\overline{T}_e \leq \frac{2}{3}\, T_r,$$

equality holding when $k\, \overline{T}_e \ll$ ionization energy.

We write n for the total number of atoms per cm³; $n_A = n\,(1 - \alpha)$ for the number of those in state A and $n_{A^+} = n\,\alpha$ for those in state A^+, so that α represents the degree of ionization. It is assumed that the ionization is nearly complete (α near unity). Let there be n' atoms A per cm³ in the low energy state, and n'' in the metastable state B, so that $n' + n'' = n_A$. If the atom A has a charge $i - 1$, the ion A^+ a charge i, the number of electrons per cm³ is given by $n_e = i\, n_{A^+} + (i - 1)\, n_A$, which for nearly complete ionization is approximately equal to $i\, n_{A^+}$.

Each ionization after some time is followed by a recombination. In each cycle the electron starts with a mean kinetic energy $\frac{3}{2}\, k\, \overline{T}$; it returns with a mean kinetic energy $\frac{3}{2}\, k\, \overline{T}_e$; in the mean time the electron must have lost the difference. If there are N ionizations per cm³ per second, the total energy lost by the electrons per cm³ and per second amounts to:

$$\frac{3}{2}\, N\, k\, (\overline{T} - \overline{T}_e).$$

The quantity N is equal to n_A multiplied by a complicated factor depending on the spectrum and the intensity of the ultraviolet radiation and on atomic constants; this factor, however, does not depend on T_e. It can be considered as a function of T_* or of $\overline{T}$ and can be treated as a constant for a given physical situation.

The electrons lose their energy in consequence of collisions with the atoms, in which the atoms are excited from the ground level to the metastable level (the energy stored in this way in the atoms is afterwards radiated away into space and thus is lost). The number of exciting collisions F per cm³ and per second is proportional to the product of n' and n_e, together with a factor depending on atomic constants and on T_e, in such a way that one can write:

$$F \sim n'\, n_e\, T_e^{-1/2}\, e^{-\chi_{AB}/kT_e}.$$

In a state of equilibrium we now must have:

$$\frac{3}{2} N k (\overline{T} - \overline{T}_\varepsilon) = F \chi_{AB}.$$

When the expression for N is introduced and when it is assumed that n'' is small compared with n', so that we may write $n' \backsimeq n_A$, the number n_A drops out and there remains an equation for the electron density, which can be written in the form:

$$n_\varepsilon = C (\overline{T} - \overline{T}_\varepsilon) T_\varepsilon^{1/2} e^{\chi_{AB}/kT_\varepsilon},$$

where the "constant" depends on atomic properties and on the incident radiation.

Since the density of the gas is given by

$$\varrho = n m = (n_{A^+}) m = (1/i) n_\varepsilon m$$

(m being the mass of an atom) and the pressure by

$$p = (n_\varepsilon + n_{A^+}) k T_\varepsilon = (1 + 1/i) n_\varepsilon k T_\varepsilon,$$

we find:

$$\varrho = \frac{Cm}{i} (\overline{T} - \overline{T}_\varepsilon) T_\varepsilon^{1/2} e^{\chi_{AB}/kT_\varepsilon}$$

$$p = \left(1 + \frac{1}{i}\right) C k (\overline{T} - \overline{T}_\varepsilon) T_\varepsilon^{3/2} e^{\chi_{AB}/kT}.$$

With appropriate units of mass and pressure, depending on the physical situation, a unit of temperature equal to $\chi_{AB}/k = 21{,}800°$ for [N II] and $\overline{T}_\varepsilon = \frac{2}{3} T_\varepsilon$, the equation of state for the gas can be put into the form:

$$\frac{1}{v} = \varrho = T_\varepsilon^{1/2} \left(\overline{T} - \frac{2}{3} T_\varepsilon\right) e^{+1/T_\varepsilon};$$

$$p = \frac{T_\varepsilon}{v} = T_\varepsilon^{3/2} \left(\overline{T} - \frac{2}{3} T_\varepsilon\right) e^{+1/T_\varepsilon}.$$

The model of a nebula consisting of a single substance A (nitrogen in the present example; oxygen would also be possible) has been given to bring out the essential features in a simple treatment. A case which approaches more to an actual nebula is a gas consisting of hydrogen with some amount of the substance A admixed to it; in that case the electrons which excite the forbidden lines of nitrogen or oxygen are mainly furnished by the hydrogen. It can be shown that the same

equations hold approximately, provided that the ionization of the hydrogen and of the substance A is nearly complete.

For a given liberation temperature $\overline{T}$, both the volume per unit mass v (or the density ϱ) and the gas pressure p are therefore functions of a single parameter T_e. For a given physical situation, and not too large absorption, $\overline{T}$ is constant. One can now draw curves of constant liberation temperature $\overline{T}$ as are given in fig. 1. It is found that, for $\overline{T}$ larger than a critical value $\overline{T}_{cr} = 1.968$, separation in two phases of equal pressure occurs. For instance with $\overline{T} = 4.5 = 98{,}000°$ (stellar temp. $\cong 147{,}000°$) one has for the dilute phase 1 and the dense phase 2:

$$v_1/v_2 = \varrho_2/\varrho_1 = 15,$$

$$T_{e1} = 5.1 = 110{,}000°,$$

$$T_{e2} = 0.35 = 7500°.$$

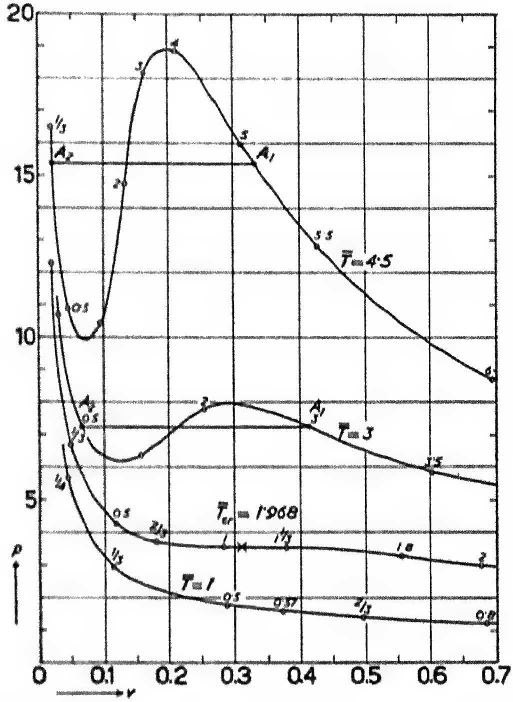

Fig. 1. p-v-diagram with curves of constant liberation temperature, $\overline{T}$.

On the basis of these results the condensations observed by Baade in the *planetary nebula in Aquarius* NGC 7293 were discussed.

I should like to make a remark on another case of condensations, viz. the filamentous condensations in the *Crab nebula* [2,3]. The electron temperature in the main body of the Crab nebula, the medium, must be very high, at least $100{,}000° = T_{e1}$, while in the condensations, where the forbidden lines are observed, the electron temperature is certainly low, of the order 5000°. If one assumes $T_{e1} = 250{,}000°$ and $T_{e2} = 5000°$, one should have, for equality of pressure, $\varrho_2/\varrho_1 = T_{e1}/T_{e2} = 50$. If the concentration in the main body is 10^3 particles per cm^3, one should have for the condensations 5×10^4 per cm^3, which is perhaps not unreasonable.[4]

Two possible excitation mechanisms have been put forward. One can assume one of the central stars to be responsible, which would require a stellar temperature of about 400,000°.[3] On the other hand, since the Crab nebula expands with a velocity of about 1000 km/sec, the nebula might entirely be excited in consequence of collisions with the interstellar gas.[5] Assuming, for the sake of argument, that for each ionization the average energy of the electrons upon liberation is 30 electron volts, one would have $\overline{T} = 232{,}000°$. The ratio $v_1/v_2 = \varrho_2/\varrho_1 = 50$ occurs for the curve of constant liberation temperature, $\overline{T} = 10$ units, so that the unit of temperature would be about 23,200°, which is of the right order. Moreover the theory would require $T_{s1} = 12.5 = 290{,}000°$ and $T_{s2} = 0.23 = 5300°$, in approximate agreement with the above estimates.

A question which might be raised is: Is the filamentary structure of the condensations in the Crab nebula itself bound up with the collision with the interstellar gas?

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CHAPTER 14

THE FORBIDDEN LINE SPECTRA OF GASEOUS NEBULAE

BY

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The spectra of many gaseous nebulae contain forbidden emission lines resulting from transitions between low-lying metastable levels of various ions (O^+, O^{++}, N^+ etc). From a study of the relative intensities of these lines it is possible to obtain important information about the physical conditions in the nebulae. In the present note we consider the determination of electron temperatures T_e and electron densities N_e.

The relative intensities are determined by three processes: excitation by electron impact, deactivation by electron impact and by spontaneous radiative transitions. We consider first the simplest case, an ion with ground state A and one metastable state B. The equation of equilibrium is

$$N_B (A_{BA} + \alpha_{BA} N_e) = N_A \alpha_{AB} N_e, \tag{1}$$

where:

> N_A, N_B are the number of ions per cm³ in each state,
> A_{BA} is the radiative transition probability,
> α_{AB} is an electron excitation coefficient and
> α_{BA} is an electron deactivation coefficient.

The excitation and deactivation coefficients are related by the equation

$$\alpha_{AB} = \frac{\omega_B}{\omega_A} \alpha_{BA} e^{-E_{BA}/kT_e}, \tag{2}$$

where the ω's are statistical weights and E_{BA} is the excitation energy of state B. The deactivation coefficient may be expressed in the form

$$\alpha_{BA} = 8.54.10^{-6} \frac{\Omega(A,B)}{\omega_B T_e^{1/2}} \ (\text{cm}^3 \text{ sec}^{-1}). \tag{3}$$

The *collision strengths* Ω (A, B), which may be regarded as constants for positive ions, have been calculated for a number of cases of interest [1]. They are believed to be correct to within $\pm$ 40%.

The quantum emission rate per cm^3 per sec is given by

$$Q_{BA} = A_{BA}\, N_B \tag{4}$$

and using (1) and (2) this may be written as

$$Q_{BA} = N_A \frac{\omega_B}{\omega_A} e^{-E_{BA}/kT_e} \left(\frac{A_{BA}\, a_{BA}\, N_e}{A_{BA} + a_{BA}\, N_e}\right). \tag{5}$$

It is convenient to introduce a critical value of N_e defined by

$$N_e^c = A_{BA}/a_{BA}. \tag{6}$$

If $N_e \ll N_e^c$,

$$Q_{BA} \simeq N_A \frac{\omega_B}{\omega_A} e^{-E_{BA}/kT_e}\, a_{BA}\, N_e, \tag{7}$$

giving an emission rate equal to the excitation rate and independent of A_{BA} (deactivation being negligible). However, if $N_e \gg N_e^c$,

$$Q_{BA} \simeq N_A \frac{\omega_B}{\omega_A} e^{-E_{BA}/kT_e}\, A_{BA}, \tag{8}$$

which is the Boltzmann distribution case. When (8) is applicable, most excitations are followed by deactivation rather than by radiation, and the quantum emission rate is limited by the value of A_{BA}. Thus (8) implies, in general, an emission rate which is small compared with that for less strongly forbidden lines.

In practice we have to consider ions with two metastable levels, B and C. In order to illustrate the essential features of the equations in their simplest form we neglect all B$\rightarrow$C and C$\rightarrow$B transitions. For practical problems this is, of course, not justified. In this approximation the ratio of the emission rates is given by

$$\frac{Q_{BA}}{Q_{CA}} = \frac{A_{BA}}{A_{CA}} \cdot \frac{a_{AB}}{a_{AC}} \left(\frac{A_{CA} + a_{CA}\, N_e}{A_{BA} + a_{BA}\, N_e}\right). \tag{9}$$

Taking levels B and C such that $E_{BA} < E_{CA}$, we always find that $A_{BA} \ll A_{CA}$, but that a_{BA} and a_{CA} are of comparable magnitudes. We may now distinguish three cases in order of increasing N_e: —

Case (1) $A_{CA} \gg \alpha_{CA} N_e$ and $A_{BA} \gg \alpha_{BA} N_e$. We then have

$$\frac{Q_{BA}}{Q_{CA}} \simeq \frac{\Omega(A, B)}{\Omega(A, C)} \, \epsilon^{(E_{CA}-E_{BA})/kT_e}, \tag{10}$$

which is independent of N_e and may be used to determine T_e.

Case (2) $A_{CA} \gg \alpha_{CA} N_e$ but $A_{BA} \ll \alpha_{BA} N_e$. We then have

$$\frac{Q_{BA}}{Q_{CA}} \simeq \frac{\Omega(A, B)}{\Omega(A, C)} \, \epsilon^{(E_{CA}-E_{BA})/kT_e} \left(\frac{A_{BA}}{\alpha_{BA} N_e}\right). \tag{11}$$

This may be used to determine N_e if T_e is known.

Case (3) $A_{CA} \ll \alpha_{CA} N_e$ and $A_{BA} \ll \alpha_{BA} N_e$. We then have

$$\frac{Q_{BA}}{Q_{CA}} \simeq \frac{A_{BA}}{A_{CA}} \frac{\omega_C}{\omega_B} \cdot \epsilon^{(E_{CA}-E_{BA})/kT_e}, \tag{12}$$

which is the Boltzmann case, for which the emission rate rate is again independent of N_e.

We consider the application of these methods in various astrophysical problems.

Planetary nebulae. The values of T_e may be obtained from the [O III] lines, for which *case* (1) is applicable. Recent work [2,3] suggests that the temperatures are both somewhat higher and also somewhat less uniform than was believed earlier [4] (it appears that T_e ranges from 1.10^4 °K to at least 2.10^4 °K in typical planetaries).

As an example of the use of relative forbidden line intensities to determine N_e we may consider planetary nebula NGC 7027, for which the observational data is particularly rich [5]. We adopt a temperature of $T_e = 2.10^4$ °K; the calculated values of N_e are not very sensitive to the value of T_e assumed. Errors in the intensities arising from differential space absorption have been eliminated by multiplying by a factor depending on wavelength which brings the observed hydrogen intensities into agreement with the theoretical values.

The following Table gives the values of N_e obtained from various ions. For N II, S II and O II equations similar to* (9) have been used (the conditions for these ions come close to *case* (2)). The value marked "O/N" has been obtained from [N I], [O I], [N II] and [O II] lines,

* The equations used [3] include all B → C and C → B transitions.

assuming that the O I/N I abundance ratio equals the O II/N II abundance ratio:

NGC 7027

Method	$10^{-4}N_e$
O II	4.4
S II	2.8
N II	5.4
O/N	5.2
Surface Brightness	0.94

The last figure is obtained by an entirely different method [6], the surface brightness in the Balmer continuum. For this method it is necessary to know the absolute surface brightness and the absolute dimensions of the nebula. Since these quantities are known less reliably than the relative intensities, it is possible that some of the discrepancy between the two methods is a consequence of observational error*. It appears probable, however, that at least part of the discrepancy is real.

The two methods could only be expected to give identical results if the density were uniform. If it is not uniform differences in the calculated values of N_e arise from different methods of averaging. The surface brightness method gives essentially a root mean square average over a specified geometrical volume, while the forbidden line methods give an average for the denser clouds which may be responsible for most of the emission. One method by which such dense condensations could be formed has been discussed by Professor Zanstra at this conference. If all the matter were concentrated in clouds of uniform density, we could estimate from the above figures that only some 5% of the total volume would be occupied by clouds.

The Orion Nebula. Similar methods may be used for the bright central regions of the Orion nebula [7]. The following values have been obtained (taking $T_e = 1.2.10^4$ °K):

The Orion Nebula

Method	N_e
S II	$1.0.10^4$
O/N	7.10^3
Surface Brightness [8]	10^3—10^4

* Note added in proof (November 1954): It now appears that the discrepancy is largely due to the fact that the brightness estimate used [6] has not been corrected for space absorption.

Nebulosity in Cassiopeia. Dr. Minkowski has shown a remarkable spectrum of the nebulosity in Cassiopeia [9]. While the [O I] lines (λ 6300, 6364) and the [O III] lines (λ 4959, 5007) are strong, the [O II] lines (λ 3727) do not appear. The suppression of [O II] is probably a deactivation effect. We give below the values of N_e^c (equation (6)) for these transitions (assuming $T_e = 10^4$ °K):

Transition		N_e^c
[O I],	λ 6300, 6364	$1.4.10^6$
[O II],	λ 3727	6.10^3
[O III],	λ 4959, 5007	9.10^5

It is seen that deactivation becomes important for [O II] at much lower densities than for [O I] and [O III].

Thus electron densities of 10^4 or greater would lead to a suppression of [O II] relative to [O I] and [O III] (for example, with $N_e \sim 10^6$ and equal ionic abundances, [O II] would be fainter than [O I] and [O III] by a factor of order 100). Thus high electron densities ($N_e > 10^4$) appear to provide the most natural explanation of this spectrum.

*Nova Shells**. Strong [O I] emissions are observed in the early stages of novae (the O I flash). The intensity ratio $I\,(6300+6364)/I\,(5577)$ at first remains nearly constant, and then begins to increase. At a later stage the [O III] lines become strong (the O III flash) and [O I] dies away. The ratio $I\,(4959+5007)/I\,(4363)$ is again at first nearly constant and then begins to increase. The [O II] lines (λ 3727) become strong only at a much later stage, when expansion of the shell has greatly reduced the electron density [13].

The most plausible interpretation appears to be that in the early stages the conditions for [O I] are close to *case* (3), giving the intensity ratio to be approximately constant despite the expansion of the shell. However, as the expansion continues *case* (2) becomes applicable and the ratio increases (the rate of decrease of density being approximately proportional to the cube of the time [11,12]). A similar sequence then occurs for [O III].

From the intensity ratios at any instant we may obtain a relation between N_e and T_e. The following Table gives this relation for Nova

* This section has been added at the time of preparing these notes for publication. The author is indebted to Dr. Minkowski for a discussion of the problem and to Dr. K. Wurm for his comments. For general references, see [10, 11, 12].

Herculis 1934, at the time when the [O I] ratio was approximately constant and at the time when the [O III] ratio was approximately constant. The lowest values of T_e given correspond to the *case* (3) limit:

Nova Herculis 1934

[O I]

$$\frac{I(6300 + 6364)}{I(5577)} = 1.7$$

T_e	N_e
$6.4.10^3$	$\gg 2.10^8$
8.10^3	$1.5.10^8$
1.10^4	5.10^7
2.10^4	9.10^6
4.10^4	3.10^6

[O III]

$$\frac{I(4959 + 5007)}{I(4363)} = 3.2$$

T_e	N_e
$9.4.10^3$	$\gg 2.10^7$
1.10^4	5.10^7
$1.5.10^4$	8.10^6
2.10^4	5.10^6
4.10^4	$2.5.10^6$

It is seen that the orders of magnitude obtained for N_e are consistent with the suppression of [O II], λ 3727. They also appear to be consistent with estimates made from a consideration of the total mass and dimensions of the shell.

[N II] lines. Dr. Minkowski[9] has drawn attention to the fact that the [N II] lines (λ 6548, 6584) are enhanced relative to Hα in filamentary structures. It does not appear possible to explain this in terms of any peculiarities of the excitation cross sections. It is possible that the effect is a consequence of changes in the ionization equilibrium with changes in electron density. In low density regions most of the nitrogen may well be doubly ionized. In this case the higher densities in the filaments would give an increase in N^+ abundance relative to proton abundance, and hence an increase in [N II] intensities relative to hydrogen intensities.

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CHAPTER 15

DISCUSSION ON CONDENSATION AND TEMPERATURE REGULATION IN INTERSTELLAR GAS CLOUDS

Chairman: Dr. E. C. BULLARD

SEATON: I would like to make some remarks on the theory of the formation of condensations proposed by Professor Zanstra.

Consider the hypothetical nebula with only two ions, A and A^+, and assume that A has a low-lying metastable level but that A^+ has not. Denote the densities by $N(A)$ and $N(A^+)$, and denote by I the energy radiated, per cm^3 per sec, in transitions from the metastable state of A.

Two conditions must be fulfilled if condensations are to grow by Zanstra's method:

(i) $N(A^+) \gg N(A)$. If this is satisfied, $N(A)/N(A^+)$ is proportional to the density and the efficiency of cooling may become greater in condensations.

(ii) $I \backsim N(A) \cdot N_e$, where N_e is the electron density. This is satisfied if deactivation is unimportant. It requires that N_e should be less than a certain critical value, which gives an upper limit to the densities which can result from Zanstra's mechanism.

In certain planetary nebulae there is some evidence for the formation of condensations. It is therefore of interest to ask if conditions corresponding to (i) and (ii) are satisfied. The ion mainly responsible for the cooling is O^{++}. It is probable that a good deal of the oxygen exists as O^{+3}, O^{+4} etc, which have no low metastable levels. If this is so, condition (i) may be regarded as satisfied.

The values of N_e in the condensations may be of order $5.10^4/cm^3$. The critical value for O^{++} is of order 10^6. It therefore appears that condition (ii) may also be satisfied and that Zanstra's mechanism may play an important role in forming condensations in planetary nebulae.

The situation is complicated by the fact that an increase in density

will tend to increase the abundance of O^+, for which the critical density is very low ($\sim 5.10^3/cm^3$). Due to this effect it is possible that densities of order $5.10^4/cm^3$ are in fact close to the upper limit which can be obtained by this mechanism.

MINKOWSKI: I would first make a few remarks on condensation in certain nebulae, and observe that before going into too many speculations it is good to go back once more to the observational data. The occurrence of condensations is quite different in different types of nebulae:

1. Some are marked by the extreme absence of condensations. Example: the Owl nebulae NGC 3587. It looks very much the same in different colors.

2. Some show a mottled appearance, particularly in Hα photographs. Example: NGC 1501. This particular nebula has the same appearance in Hα and [O III]. In this case, the structure seems to show directly an uneven distribution of density.

3. Some are very peculiar and the nebulae NGC 7293 is an example. The tiny condensations in it have radial extensions that resemble comet tails. They are quite sharp on a red plate (mainly [N II]), but more diffuse on a plate taken in the light of [O III]. This shows that the mass-concentration is not as the red plate suggests.

In general, it is an empirical law that the [N II] lines react very sensitively to conditions of excitation. They can be enhanced in sharply defined local favorable areas in a large diffuse object, and show then the distribution of a peculair condition of excitation, not the true mass distribution.

My further remark refers to Seaton's discussion, by which I am not quite satisfied. What is the essential difference between the O^+ and the N^+ ion? In ordinary nebulae sometimes [O II] is stronger and sometimes [N II] without significant differences in the rest of the spectrum.

SEATON: The intensity ratio of [N II] λ 6548, 6584 to [O II] λ 3727 depends on electron temperature, electron density and the relative abundance of N^+ and O^+ ions, all of which may be variable. [O II] λ 3727 differs from most other forbidden lines in that it has a much lower radiative transition probability. The electron density for which radiation and deactivation are equally probable is 6×10^3 cm^{-3} for [O II] and is 1×10^5 cm^{-3} for [N II]. My point about the Cassiopeia source was that if [O I] and [O III] are present, then there must be a considerable density of O^+ ions. The absence of [O II] λ 3727 then indicates that the electron density must be much greater than 6×10^3 cm^{-3}.

MENZEL: I have been developing a theory of solar prominences. A filament of conducting gas, descending through a magnetic field, will have induced in it an electric field. Under some circumstances a discharge can take place and a current build up of the order of 10^8 to 10^9 amperes, within an interval of minutes. Inductance rather than conductivity controls the rate of build-up of these currents. Two types of forces are involved. One causes a loop to form, which expands much like a smoke ring. At the same time, the so-called "pinch effect" tends to reduce the cross section of the current-carrying element. This simple prominence model accounts for many of the observed properties of prominences.

I have been considering the possibility that the filamentary structure of gaseous nebulae may also arise from the pinch effect, or possibly from a combination of this effect with those described by Zanstra. On a still larger scale, one may speculate that the spiral arms of galaxies may also have come into operation through the action of electric currents and the associated pinch effect.

GOLD: Kahn's discussion of the heating of clouds in a collision between two clouds should be completed by including the magnetic pressure. This important effect may cause a more or less elastic collision, so that less energy is liberated than Kahn computes.

SCHATZMAN: How many clouds are actually in the process of collision? Would it be possible to recognize them?

KAHN: Some five or ten per cent of the clouds may be in collision, but I do not see how to distinguish them.

BOK: Kahn's estimates should be different for regions in a spiral arm or between arms. For it is clear, e.g. from Münch's observations, that the H II clouds, as well as the neutral hydrogen, are concentrated in the arms.

OORT: All estimates made so far refer to the region near the Sun, which is in a spiral arm. As an answer to Schatzman I would suggest that perhaps the nebulae near Merope with their wave structure are a case of colliding clouds.

KAHN: Can Savedoff tell me how low the temperature of the interstellar gas can possibly become, if the cooling effect of the hydrogen molecules is added?

SAVEDOFF: This is still of the order of 50° (as in the Spitzer-Savedoff papers).

PART III

SHOCK WAVES AND COLLISION PROBLEMS

CHAPTER 16

EXPERIMENTS ON THE RADIATION AND IONIZATION PRODUCED BY STRONG SHOCK WAVES

BY

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Ithaca, N.Y.

I want to begin by reviewing for you some of the well-known qualitative aspects of shock waves. I do this in the hope that we may be more readily able to determine whether any celestial observations reveal similar phenomena. To my mind, the most striking property of shock waves is their tendency to produce a certain amount of macroscopic order while they are producing a large amount of molecular disorder.

A first illustration of this organizing tendency is most readily brought out if we consider the formation of a shock from an arbitrary compression wave in a one-dimensional process. The forward elements of the wave are overtaken by the rearward elements as a consequence of two phenomena. First, the forward elements create a gas motion which transports the rearward elements in the propagation direction, thus increasing their propagation velocity. The propagation velocity of the rearward elements is generally increased also by the fact that the medium is heated by the previous compression process. These effects are additive so that under almost all circumstances compression waves will steepen up in time until the pressure shows an almost vertical step. The steepness is finally limited by transport phenomena which occur when the thickness of the compression wave becomes comparable with the mean free path. Compression waves, which have attained this degree of steepness are called shock waves. Thus, the process of formation of shock waves organizes arbitrary compression disturbances into "rough" step functions.

Another characteristic feature of shock waves is their tendency to form smooth surfaces. Thus a concentrated curvature at any point on a shock wave diffuses out so that the curvature tends to equalize with that of nearby regions. When a shock wave propagates down a straight

pipe as in a shock tube, boundary conditions require that the edge of the shock be normal to the tube walls. The diffusion of any initial curvatures then results in the shock becoming accurately plane and perpendicular to the tube axis. Again in explosions and implosions where the shock forms a closed surface, the equalization of curvature results in the formation of spherical or, in two dimensions, cylindrical shock waves. These organizing aspects should provide considerable aid in the identification of celestial observations with shock phenomena. To complete such an identification we must show that the mass velocity of the gas and its thermodynamic state changes abruptly in a distance of the order of two mean free paths. We should also expect celestial shock waves to exhibit much smoother geometrical contours than the gas clouds in which they form.

In the identification of shock waves and in determining the role of shock phenomena in astrophysics, laboratory studies of strong shock waves can make a contribution. My primary object in this lecture is to present a review of studies of this kind which have been in progress at Cornell University for a number of years.

The first objective in this program has been the development of techniques for the production of high temperature gases under laboratory conditions which would permit close control of the gas state produced. For this purpose we have extended the application of the shock tube (Fig. 1) in two ways. First, we have employed large pressure ratios across the diaphragm to produce strong shocks. When large pressure ratios are employed, the shock velocity U_s which can be obtained is directly proportional to the speed of sound in the driver gas. In Fig. 2 we present theoretical shock speeds obtained from conventional shock tube theory for various ratios of the speed of

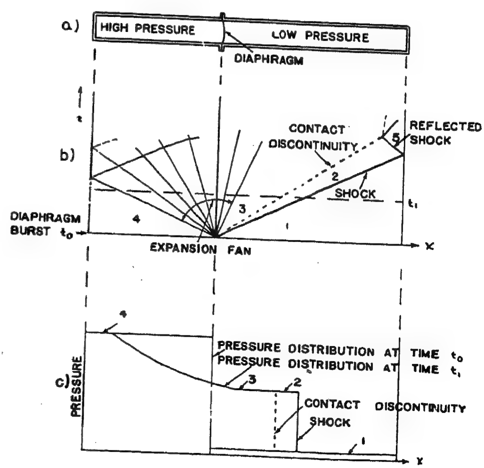

Fig. 1. Schematic representation of a shock tube.

(a) shows a shock tube. Note that the diameter to length ratio has been exaggerated.
(b) the position of various phenomena involved in the production of the strong shock versus time.
(c) the pressure distribution at the instants t_0 and t_1 shown in (b).

sound in the (diatomic) driver gas a_4 to the speed of sound in a monatomic driven gas a_1. In many of our experiments these gases were hydrogen and argon respectively and with this combination Mach numbers as high as about 12 are attainable using pressure ratios of the order of 10^4. For the attainment of higher shock speeds we have added a small amount of oxygen (about 16%) to the hydrogen, thus after combustion we attain a sound speed roughly 60% higher than that of pure hydrogen. This permits the attainment of Mach numbers greater than 17.

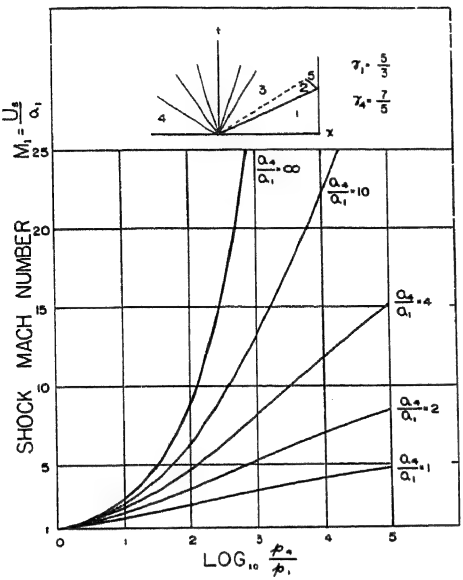

Fig. 2. Relation between the shock Mach number and the diaphragm pressure ratio. Curves are plotted for various ratios of the velocity of sound in the high and low pressure gases.

The apparatus in which most of these shock experiments have been carried out is shown in Fig. 3. The shock velocity produced is measured employing spark plugs (to detect the conductivity produced by the shock) distributed at known stations along the shock tube in a standard chronograph procedure. Alternatively the shock speed was measured employing the luminosity of the shock phenomena and a drum camera. It is found that the shock produced is of constant strength as measured by the speed and that the speed is roughly 10% lower than that calculated from the results of Fig. 2. The technique of the production of strong shocks was developed in collaboration with Dr. E. L. Resler and Dr. S. C. Lin [1].

This technique has two important advantages over other methods for the production of high temperature gases for aerodynamic studies. In the first place, from measurement of the shock velocity an accurate value of the enthalpy of the gas immediately following can be obtained. If we write conservation of energy across the shock wave

$$H_1 + \frac{u_1^2}{2} = H_2 + \frac{u_2^2}{2}, \tag{1}$$

where H is the enthalpy per unit mass and u is the gas velocity measured

relative to the shock wave and conditions 1 and 2 are conditions ahead and behind the shock wave. For strong shocks the enthalpy of the gas ahead of the shock wave H_1 is negligible compared to the enthalpy behind. Combining eq. (1) with the equations of conservation of matter, of mass and momentum, it can readily be shown that the kinetic energy of the gas behind the shock is negligible compared to the enthalpy behind the shock. Thus to a fair approximation (within 5% for $M > 10$) we have

$$H_2 = \frac{u_1^2}{2}. \tag{2}$$

This equation brings into prominence the fact that the enthalpy of the gas behind the shock is determined almost entirely by the gas velocity ahead of the shock as measured in shock coordinates. This in laboratory coordinates is merely the shock velocity U_s so that a shock velocity measurement (which can be made with great accuracy) gives an immediate value of the enthalpy behind the shock.

Another important advantage of the technique is that since the high temperatures are produced by shock waves and other aerodynamic phenomena it will be possible directly in the shock tube to study some of these phenomena. For example, the processes which accompany the transfer of energy from the mass motion of gas to thermal energy in shock waves can be studied in the shock tube. It will be expected that these processes will be considerably different from the equilibration processes which occur in arcs and other high temperature electrical phenomena.

To summarize the present state of this technique it is now possible to produce highly ionized gases in shock tubes and to have an accurate estimate of the gas enthalphy. The enthalpies which have been produced are sufficient to ionize argon 20% ($M = 17$) or after reflection from a closed shock tube end to ionize argon 50%. It is thus possible in a shock tube to produce highly ionized gas in violent motion and to make studies of the dynamics of such gases. Our first studies have been made utilizing a monatomic gas to avoid the complexities and energy loss inherent in dissociation processes which would occur in diatomic or polyatomic gases. Of the monatomic gases argon is the obvious choice since its high molecular weight permits the achievement of large molar enthalpies. Another advantage of argon is that a great deal of information concerning its atomic properties is available.

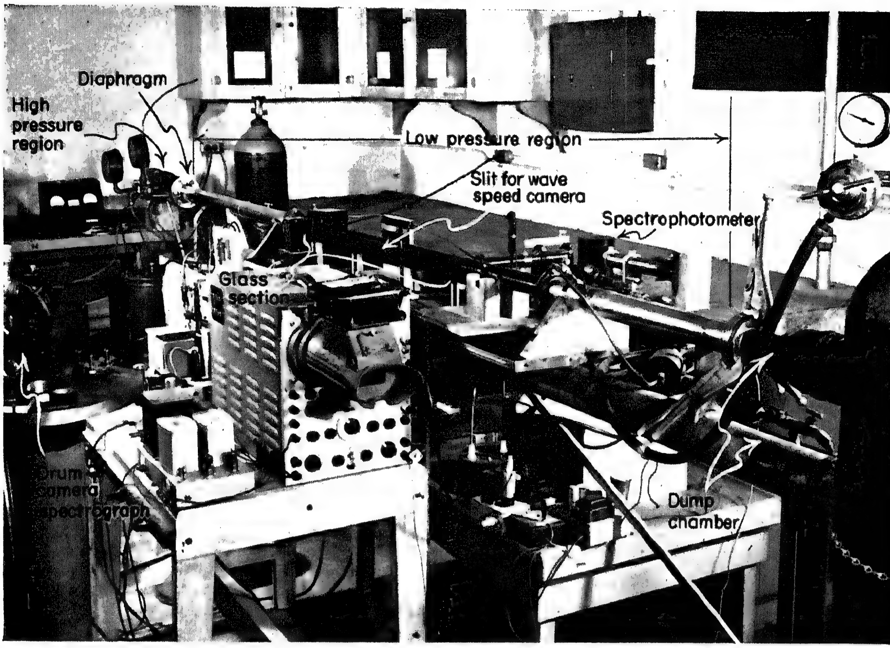

Fig. 3. Apparatus for the production of strong shock waves

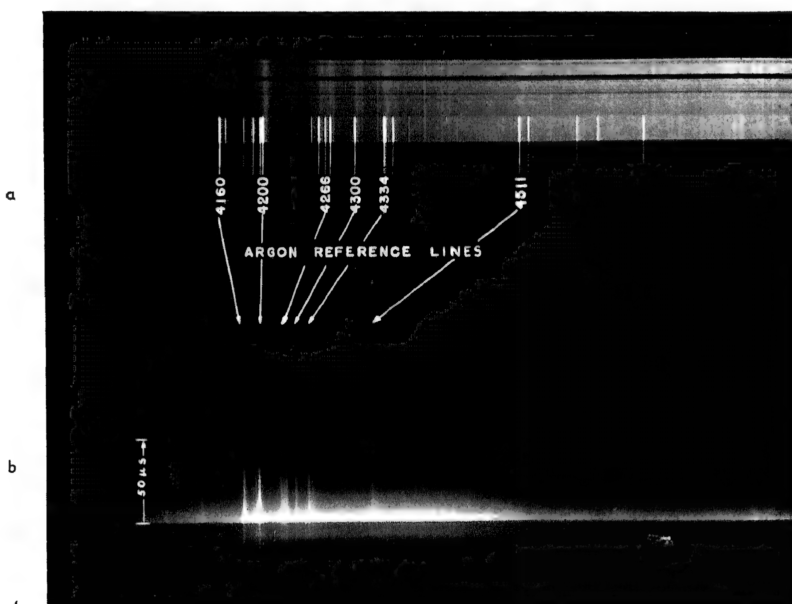

Fig. 4.

(a) An enlargement of a small portion of a spectrum (Argon Geisler tube comparison). It can be seen that the shock argon lines are greatly broadened and shifted to the red.

(b) An enlargement of a similar portion of a spectrum taken with a drum camera

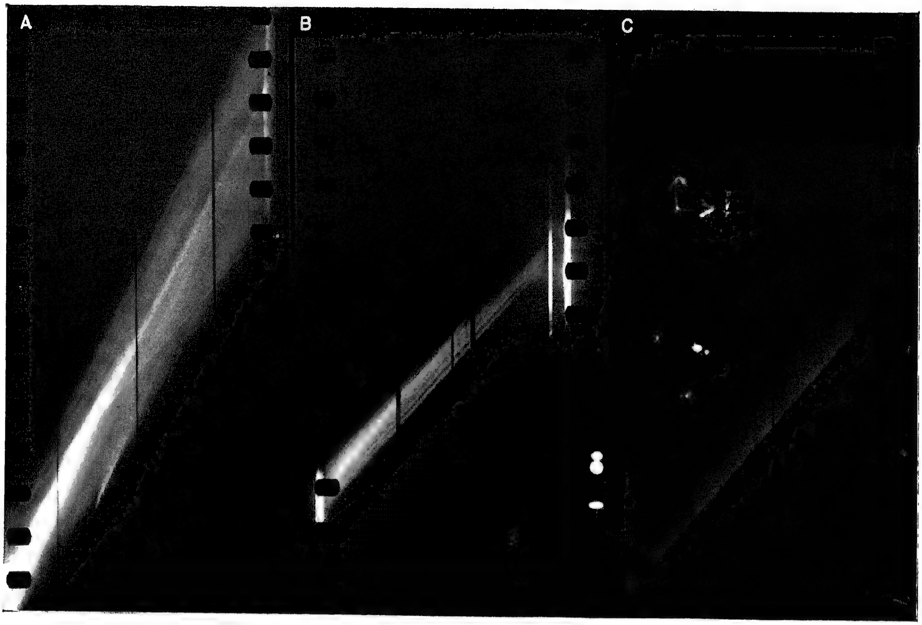

Fig. 8. Drum camera photographs of the luminosity produced by shock waves. Time increases upwards. The vertical dark streaks are produced by pieces of black tape wrapped around the shock tube.
In (A) it will be noticed that the luminosity increases gradually following the shock wave, which can be identified from the beginning of particle paths. It will be noticed that in the luminous region a second shock wave is to be seen, which can be found from the higher velocity as compared with particle paths. It will be noticed that this shock is self-luminous; that is, it is more luminous than the gas which follows. The Mach number of this shock is 8.43 and it was produced by a combustion driver.
In (B), we show a shock, Mach Number 7.7, produced using a pure hydrogen driver. In this case, it is noticed that the shock itself is luminous and is followed by a markedly less luminous region. The luminosity which follows sometimes later is produced by particles which originated in the metal portion of the shock tube.
In (C), we show a Mach Number 10.9 shock produced by a combustion driver, in which quite different behaviour is recorded. In this case the luminosity appears suddenly and persists thereafter. The gradual reduction in intensity can readily be interpreted as radiation cooling of the ionized gas.

SPECTROSCOPIC STUDIES

It has been known for several years that a tremendous amount of light is given off from strong shock waves, especially in monatomic gases. This suggests the use of spectroscopy to study the internal state of the gas. In particular, it was desired to determine how closely the gas approached thermal equilibrium. As the work progressed it was found possible to make time resolved spectra and absolute spectrophotometric measurements so that rate processes in the high temperature gases could also be studied.

In our first spectroscopic experiments it was found that the spectra were dominated by easily ionizable materials. The characteristic transition from the spectra of unionized to that of singly ionized alkaline earths as the temperature was raised was very prominent. However, the relative line strengths could not readily be used to study the gas state in our case since the concentration of impurities was not known. Using temperatures over 10,000 °K it was readily possible to produce argon spectra when the impurity levels were kept to about 1%.

An enlarged portion of such a spectrogram is shown in Fig. 4a, together with a Geissler tube argon reference spectrum. It will be seen that the argon lines in the shock spectra are displaced several Ångströms to the red and that they are considerably broadened. In order to study the variation of the line shift with time following the passage of a shock wave, a drum camera was constructed which could be used together with a grating to furnish a time-resolved spectrogram. Such a spectrogram is shown in Fig. 4b. It will be seen that the line broadens almost immediately following the onset of luminosity and that thereafter the line broadening and shift decays rapidly. We interpret this as meaning that equilibrium is rapidly approached in the gas and thereafter the ion density reduces due to cooling.

This broadening and shift was attributed to Stark effect produced by the fields of neighboring positive ions and electrons. The effects of the positive ions (using known Stark coefficients) could be treated using an analysis due to Holtsmark. The effect of the electrons however could not be regarded as due to their static field as in Holtsmark's treatment but had to be treated essentially as collision phenomena in the manner of the Lorentz and other treatments of collision broadening. This treatment was carried out by Dr. Marcel Baranger with the help of Prof. H. Bethe. It was found that the line contours obtained experimentally were in agreement with Baranger's treatment.

The theoretical line shifts vary nearly linearly with the ion density. Therefore, accepting the theoretical treatment, the line shifts can be used to measure the degree of ionization in argon since the Stark coefficients are known. Assuming thermal equilibrium, it is possible to

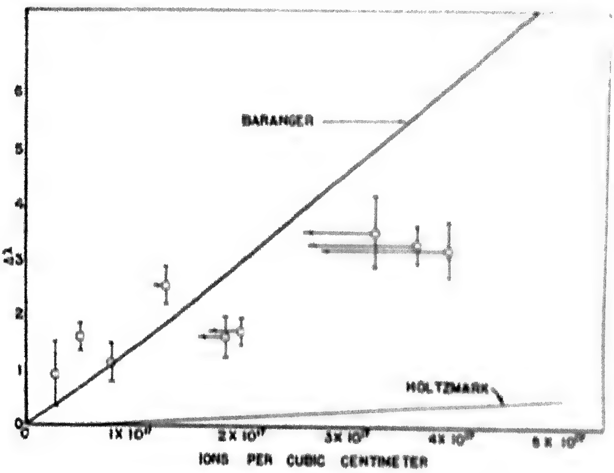

Fig. 5. A comparison between the line shifts measured and calculated. The ion density in the gas is calculated from the Saha equation employing the known enthalpy from shock speed measurements. The values plotted here are obtained from drum camera spectrograms.

calculate the degree of ionization from the measured shock velocity. The shock velocity gives the enthalpy of the gas directly as was shown above and then the degree of ionization can be obtained from the Saha equation. The experimental line shifts are plotted against this calculated ion density in Fig. 5. A correction to the calculated ion density, shown by the arrows, was made for cooling of the gas during the resolving time of the drum camera spectrograph. The theoretical line shifts as obtained from the Holtsmark and Baranger theories are also shown in Fig. 5. The good agreement of these experimental line shifts with the theoretical values indicate that equilibrium was obtained very quickly in the gas.

It will be noted that a strong continuum radiation is evident in the spectra of Fig. 4. This continuum is due to recombination of positive ions and electrons. Calculations of the continuum intensity for application to the radiation from arcs has been made by Unsöld, essentially amplifying the classical treatment of Kramers. To establish the source of this radiation a series of spectrophotometric measurements were made on the continuum and compared with the theory of Unsöld. It was found

that the continuum intensity was independent of wave length as expected, and that the temperature variation was in good agreement with Unsöld. In Unsöld's theory an unknown effective nuclear charge appeared and the use of unit nuclear charge fitted the experimental absolute intensity (assuming equilibrium ionization). A report on our spectroscopic work has been prepared and is shortly to be sent to the Journal of Applied Physics for publication[8]. This work was done by Mr. H. Petschek, Mr. H. Glick, Mr. P. Rose, Mrs. A. Kane and the author.

ELECTRICAL CONDUCTIVITY

One of the phenomena susceptible of easy measurement in the shock tube is the electrical conductivity. The electrical conductivity is interesting for two reasons. In the first place it serves as an indication of the electron or ion density and if the electron or ion density be accepted as equilibrium values from spectroscopic or other studies, then the electrical conductivity can be used as a measure of the electron diffusivity in a highly ionized gas.

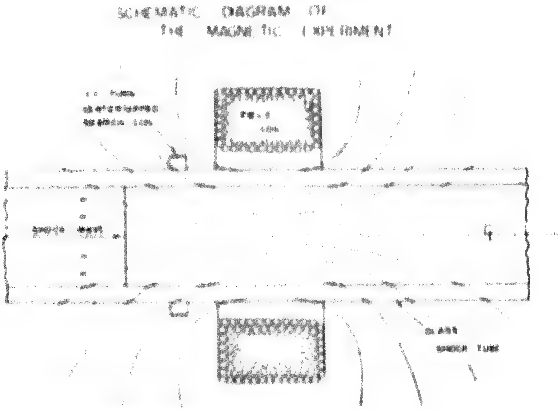

Fig. 6. Schematic drawing of the apparatus used for the measurement of electric conductivity of the high-temperature gas.
A magnetic field is created by the field coil. When the shock wave passes through the field coil this field is displaced, some of the lines cutting across the search coil. Since the field displacement depends upon the conductivity of the moving gas, it can be used to measure the conductivity. The apparatus is calibrated by moving an aluminum slug of known conductivity and known velocity through the shock tube.

A schematic drawing of the apparatus used for the measurement of electrical conductivity is shown in Fig. 6. A magnetic field is created in the shock tube by means of a flat field coil. When a shock wave passes down the tube it is followed by a highly conducting gas which displaces some of the field lines. The displacement of the field lines is measured by the voltage produced by the search coil which is directly related to the rate at which field lines cross it. The search coil voltage is recorded

on an oscilloscope and is roughly proportional to the rate of change of the conductivity with time. The apparatus is calibrated by dropping a slug of aluminum of known conductivity down the shock tube thus avoiding any necessity for any detailed knowledge of the field distribution inside the shock tube. The results of our conductivity measurements are presented in Fig. 7.

Two types of effects dominate the diffusivity of electrons. First there are the close collisions which inhibit the motion of electrons through a gas much as ordinary gaseous diffusion is inhibited by collisions. Secondly, when a gas is highly ionized the random electric fields greatly distort the electron trajectories. At high degrees of ionization (greater than 1%) these effects become predominant and the effect of close collisions becomes small by comparison. Under these conditions the electrical conductivity of the gas approaches a value which is roughly independent of the ion density. First order theoretical treatment of the electrical conductivity under these conditions has been given by Chapman and Cowling [5] and more accurate treatments have been given by Cowling [6] and by Spitzer and his co-workers [7]. These treatments differ in that a divergent integral which appears in these calculations is "cut off" in various ways. It should be noted that in these treatments the conductivity is completely independent of the characteristics of the particular gas atoms involved and is determined only by fundamental constants. Also no undetermined parameters such as cross sections appear in the theory. We have combined the close collision theory with that of Spitzer by adding the resistivities due to the two methods by which the diffusion of electrons is inhibited. This gives the resistivity curve plotted as a solid line in Fig. 7.

It will be noted that the measured conductivities are somewhat higher than the theoretical conductivities at the lowest temperatures. This is attributed to the effects of easily ionizable impurities which were present to a small extent in these experiments. At intermediate temperatures it will be noted that the conductivity is somewhat lower than the theoretical line and this is attributed to the fact that the gas did not attain full equilibrium. This lack of attainment of equilibrium can be noticed also by the detailed shape of the oscillograms in which it can be seen that the conductivity was steadily rising during the passage of the hot gas and had not attained a steady value. The high temperature results are seen to be in very good agreement with the theoretical expectations and this agreement was noticed to begin at the temperature when the conductivity stopped rising before the end of the hot gas region.

At the highest temperature the oscillograms indicated a very rapid rise to a conductivity close to the theoretical values followed by a decline in conductivity which is attributed to the rapid rate of cooling. Further study of the rate of ionization will be needed before a complete description of the ionization process can be given.

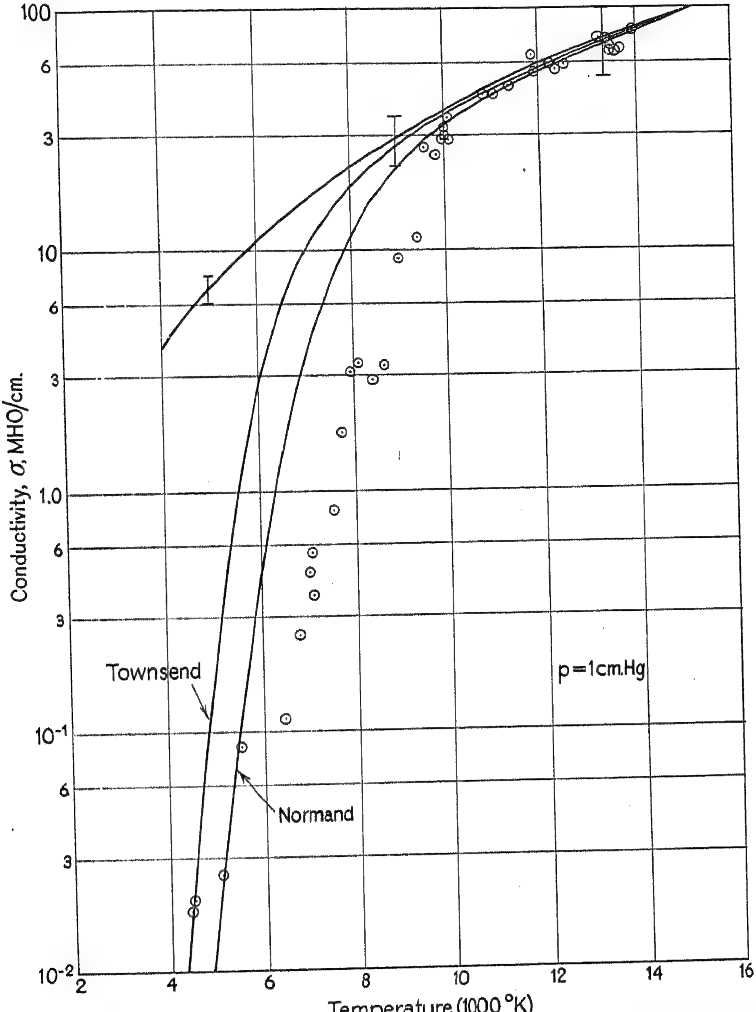

Fig. 7. Conductivity of the gas following a shock wave vs. temperature. The conductivity points shown were obtained using the apparatus diagrammed in Fig. 6. The lower solid lines show theoretical calculations which were obtained by adding the effects of inhibition of electron diffusion by ions to the usual close encounter resistivity, employing cross-sections from the work of Normand and Townsend.

This work has largely been done by Dr. S. C. Lin and Dr. E. L. Resler and will be sent for publication to the Journal of Applied Physics shortly[8].

LUMINOUS FRONT PHENOMENA

The light emission from strong shock waves in argon and in air has been studied by the following conventional technique. Employing the glass shock tube a thin slit was viewed by a wave speed (rotating drum) camera so that the waves and particle paths are recorded as on an x–t diagram (Fig. 1b). A photograph of this kind is shown in Fig. 8c. By this technique the luminosity of the gas at all times after passing through the shock wave can be studied conveniently. The result which was obtained in Fig. 8c for Mach number 10.9 is what would be expected in view of the conductivity and spectroscopic results. The luminosity builds up rapidly to the maximum value and then gradually declines corresponding to the cooling of the gas.

In Fig. 8b we present quite a different result however. In this figure the shock of Mach number 7.7 shows strong luminosity in the shock front followed by a less luminous region behind. This photograph shows in addition a luminous region following considerably later which has been found to correspond to luminous particles which were released into the gas from the region just following the diaphragm (where a metallic shock tube wall was used). The luminosity of the shock, however, is a phenomenon of special interest in view of the fact that it was found earlier that the full electronic conductivity takes a finite time to develop at these low Mach numbers and that we would expect the luminosity to be associated with a high electron density, just as the conductivity is.

Fig. 8a shows an experiment which is similar to 8b except in that the driver gas in this case was a combustible hydrogen-oxygen mixture where in the first case it was pure hydrogen. In this case no luminous front is visible at the shock wave and we do not understand the reason for the apparent difference between these two results. In Fig. 8a a bright line which crosses the particle paths can be seen. This is presumably a shock with $M = 1.4$. It is interesting that such a weak shock exhibits considerable luminosity when moving into an already ionized gas.

It has been noted that a high conductivity is sometimes associated with the luminous front itself. Conductivities far in excess of the equilibrium expectations have been found. Considerable difficulty has been found in obtaining luminous fronts consistently. Apparently the presence of condensable impurities in the gas plays an important role since the

application of cold traps seems to prevent the occurrence of the luminous front. Our present activities in this direction are concerned with experimental efforts to obtain luminous fronts consistently. Most of our work on luminous shock fronts has been done by Mr. H. Petschek and Dr. S. C. Lin.

The observation of these luminous fronts brings to mind several phenomena which may be related. In the first place in the collision of interstellar gas clouds it has frequently been noted that sharp luminous lines appear. These are especially prominent in photograps of the Crab nebula. The gas in this nebula is known to be in violent motion and it seems very likely that shock waves would occur. The luminous region in this nebula has been suggested by J. H. Oort to be the result of shock waves in the nebula. However, it was pointed out that a shock wave contains no specially hot region and therefore the high luminosity is not to be anticipated. In view of our experimental results perhaps this matter should be reconsidered.

I would like to close this lecture with a speculation regarding a possible cause of this high excitation associated with shock waves. The high mobility of electrons undoubtedly requires special treatment for inclusion in shock wave theory. One would expect for example that the electrons would readily diffuse across the shock front and that they would readily transport heat which, however, they would not share readily with the atoms but would keep to themselves as an "electronic temperature". Thus we would expect that this electronic temperature would vary only slowly in the region of the shock front and not rapidly as the gas temperature*. We would expect therefore that there could be appreciable ionization by electronic collisions ahead of the shock wave. Note now that the positive ions formed ahead of the shock wave find themselves in a strong wind tending to drag them into the hot gas whereas the electrons are much less affected by the gas velocity. Thus there is a separation effect which would tend to produce a negative charge ahead of the shock wave. If now, electrons are formed by ionization processes in a highly negative region they are therefore endowed with electrical energy by reason of their formation in this region. Calculations of the efficacy of this process show that it can produce an appreciable amount of energy but it is not clear at the present moment whether the energy production by processes of this kind is sufficient to explain the phenomena which have been observed.

* Denisse and Rocard [7] did not take this large difference between electronic and gas temperature into account in their treatment of the excitation of plasma oscillations by shock waves.

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CHAPTER 17

DISCUSSION ON SHOCK WAVES AND RAREFIED GAS DYNAMICS

Chairman: Prof. G. TEMPLE

LAPORTE: The smoothing out of irregularities in shock wave fronts is the less pronounced, the higher the number of spatial dimensions: it is slower for a spherical shock wave than for a cylindrical wave in two dimensions and it would disappear in four-dimensional space.

BATCHELOR: Is it possible to give some quantitative information concerning the rate at which irregularities are smoothed out? I had the impression that the pictures shown of luminous edges of interstellar clouds presented a rather unsmooth appearance.

LAPORTE: Quantitative data so far are not available. Shocks show a tendency to catch up with one another the faster, the stronger they are, but since cylindrical shocks decay intrinsically something like $1/r$ and spherical shocks like $1/r^2$, they will deteriorate to sonic disturbances and hence will be more and more reluctant to catch up with each other.

I would further mention the experimental work done by Fowler and co-workers at the University of Oklahoma, on shock waves produced by the sudden heating of a portion of the gas by means of a heavy condenser discharge. In this case the driving gas is hotter than the driven gas. A summary of this work can be found in a thesis by W. R. Atkison (cf. Technical Report to Office of Naval Research U.S.A., contract number Nonr 982 (02); a diagram was shown of the experimental set-up). In connection with this it is of interest to observe that Taylor has noted that a form of instability may occur when a hot gas drives a cooler gas.

The concentration of luminosity behind the shock had already been noticed by R. Becker for shock waves produced by the detonation of explosives.

TAYLOR: The instability mentioned by Prof. Laporte occurs when a deceleration immediately follows on an acceleration.

I would ask how Kantrowitz has actually measured the conductivity of the gas by means of the search coil?

KANTROWITZ: The apparatus has been calibrated by moving an aluminum object through the field.

TAYLOR: There may be difficulties caused by capacities of the glass wall of the tube in that case.

THOMAS: I would like to make a few remarks on some work we have been doing along lines similar to that reported by Kantrowitz. First, however, I think it is appropriate to emphasize two points which represent the difference in physical conditions for the work of the astrophysicist and aerodynamicist.

(1) There is the difference in the energy-level of the undisturbed medium in which one works—for the aerodynamicist the temperature of the undisturbed medium is around 300 °K; for the astrophysicist it is 10^3–10^4 °K (only for the H I-regions, in interstellar space, are the temperatures comparable). Thus the aerodynamicist requires strong shocks of the type reported by Kantrowitz (and to be reported here) to get significant radiation and to force him to worry about modifying his equations to include a variable γ. E.g., $M = 10$ just takes him near 5000°, while the astronomer starts from there, and, e.g., $M = 2$ suffices for the same concern with change in ionization, radiation, etc.

(2) Customarily, the energy transfer in aerodynamic problems is mechanical, and the radiation problems are the perturbation. The big question in most of these aerodynamic problems is whether the thermodynamic state of the gas is still fixed by the mechanical (aerodynamic) flow pattern, with the radiation playing the part of a small energy sink. Astronomically, one usually thinks in terms of radiative energy transfer—and the mechanical motions now are the perturbation, in the form of an additional energy source. Again, one wants to know how much these mechanical motions perturb the state of the gas as computed from the radiation field alone. One should remark here that there has existed too great a tendency in astronomy to consider these astro-aerodynamic effects as entering only as a force term, not altering the internal energy; i.e. only the effective pressure, and not the temperature, has been considered to need modification.

Turning then to the work proper, we have attempted to obtain some insight into these astro-aerodynamic problems by experiments in what we have called the astroballistic regime—that where a solid body radiates because of high velocity motion through a gas. The application is most directly to meteors, and the study proceeds from a joint investigation of meteor observations and laboratory experiments on artificial meteors. While the present symposium is on the interstellar medium, we have

already seen the necessity for a general orientation for *both* aerodynamicist and astrophysicist into the physical nature of the problems. Thus these experiments are in the direction of developing such physical intuition. In brief, solid bodies of dimensions of about a centimetre are propelled through the air at speeds about 5 km/sec. The actual experiments were done by J. S. Rinehart, W. C. White, W. A. Allen, E. Mayfield and are reported in J. Applied Physics (1952). White and I have been trying to interpret them, in conjunction with meteor observations. The data thus far (White is trying to extend them) give time-resolved (uncalibrated) spectra along the axis of motion. In brief, one obtains no more than 1 μsec ($\cong \frac{1}{2}$ cm) continuum at the shock front, followed by some 10–20 μsec of luminosity essentially wholly excited aluminum, the intensity of which decays rapidly. Not until $\sim 10^2$ μsec does the oxidation reaction appear. Comparing the luminosity in the excited aluminum region to the luminosity of meteors, in a very rough analysis, we concluded that atom-atom inelastic collisions are the responsible agency. Our work and that of Kantrowitz is therefore in agreement. We further conclude that conditions are very probably non-equilibrium, and we should thus be able to investigate the departures from equilibrium involved *. Such investigations are of the utmost importance in these astronomical investigations, for we do not have methods presently existing at all for treating those cases where the region of approach to equilibrium is non-trivial in extent. And these are just the circumstances likely to be of interest in astronomical application, particularly in the interstellar medium problems. I would like to emphasize here that the only investigations in this direction presently completed or under way lie, on the one hand, in the statistically steady-state investigations of the planetary nebulae and solar chromosphere, and on the other hand, in these investigations of strong shock phenomena via the artificial meteor and the shock-tube studies.

Aspects of rarefied gas dynamics

by

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Since interstellar gases are highly rarefied, it is appropriate to review very briefly some of the recent research developments in aerodynamics of rarefied gases, and in particular to list some of the more important

* The full work will be reported shortly in the Astrophysical Journal.

references. This aerodynamic work has for the most part been confined to the motion of electrically neutral gas—usually air—past solid bodies. Hence, many of the special phenomena of interstellar gas motions are not covered.

The significant parameter for indicating the relative importance of rarefaction effects is the Knudsen number, K, the ratio of molecular mean free path λ to a dimension l characteristic of the flow field (this dimension l may be a boundary layer thickness at high Reynolds number or the body diameter at low Reynolds number). Tsien[1] has characterized the various regions of flow as: continuum flow, $K < 0.01$; slip flow, $0.01 < K < 0.1$; transition flow, $0.1 < K < 10.0$; free molecule flow, $K > 10.0$.

The free molecule flow regime has been extensively investigated recently, both experimentally and theoretically. Momentum and energy transfer characteristics for a large variety of body shapes and speeds are now available[2–13]. Similar results in the slip flow range are neither so extensive nor so elegant. The essential reason for this is that the corresponding values of K, which is related to the Mach and Reynolds number by $K \sim M/Re$, are necessarily associated with either high M or small Re or both. Slip flow rarefaction effects thus occur simultaneously with extreme compressibility effects (high M) or strong viscous effects (low Re) or both. Further complications occur associated with, (1) the lack of knowledge of molecule-surface interactions and a consequent incomplete formulation of boundary conditions, and (2) the breakdown of the simple Navier-Stokes relations for the dependence of viscous stresses and heat flux on velocity and temperature gradients. These items are discussed, and some partial results are obtained[14–31]. It is appropriate to emphasize, however, the fact that rarefied flows are usually also very viscous flows.

The research program in rarefied gas dynamics which has been in progress at Berkeley is summarized[30,31]. Some aspects of this work which are particularly pertinent are:

(1) *Shock wave thickness:* Surveys of the distribution of temperature and velocity inside the shock zone have been made. It appears that the zone is approximately five upstream mean free paths thick for shocks stronger than $M \sim 2$. Available theoretical analyses[32–36], are mostly inapplicable for all but the weakest shocks, since the Navier-Stokes relations break down. A publication on this work is forthcoming.

(2) *Equilibrium temperatures:* Measurements of equilibrium temperatures indicate a marked increase in temperature with increasing rare-

faction. The equilibrium temperature becomes higher than the adiabatic stagnation temperature. This phenomenon starts in the slip flow range [26] and continues in the free molecule flow range [2,3].

(3) *Afterglow and discharge*: A technique for visualizing wind tunnel flow configurations has been developed in which the air (or N_2) has an electrical discharge passed through it exciting an afterglow state. This provides a luminous stream, so that shock waves, wakes, boundary layers, etc., are directly visible [37–39]. Since the ionization level can be controlled the possibility exists to perform wind tunnel experiments involving magneto-hydrodynamic phenomena at supersonic velocities.

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GOLD: I should like to discuss, in connection with the subject of shock waves, some of the magnetic disturbances on the earth that are caused by solar outbursts. The initial magnetic disturbance at a "Sudden Commencement" of a magnetic storm can be accounted for very roughly by an increase of pressure of the tenuous gas around the earth. This increase of pressure may perhaps be described as the effect of a wave sent out by the sun through the tenuous medium between sun and earth. In the complete absence of any such medium this description would then correspond to that of a stream of particles, while in the presence of a medium the correct description may lie anywhere between an acoustic wave, a supersonic shock wave or an unimpeded corpuscular stream.

It is known that some "Sudden Commencements" on the earth occur about 24 hours after the outburst has occurred on the sun. Thus we know the velocity of the phenomenon which propagates itself, and this velocity is of the order of twenty times or more the velocity of sound in the medium. Although we have a travel time of 24 hours or more, the initial increase in the magnetic force may reach a maximum in as little as two minutes. If this were to be attributed to a stream of particles which is unimpeded until it reaches the neighbourhood of the earth, then it would be necessary for this stream to have a quite unreasonably small velocity dispersion. Even the purely thermal velocity dispersion would cause a time of build-up of more than one half hour, and in addition it is hard to suppose the accelerating mechanism at the sun to impart such high and yet precisely identical velocities to each particle. A much more reasonable interpretation of the phenomenon would be the arrival of a highly supersonic shock wave with the characteristic sharp wave front. The effects of the thermal and other velocity dispersions would then be kept in check during the entire travel by the tenuous medium through which this wave or stream is propagating.

The observations of magnetic storms may hence give us a fairly direct proof of the existence of shock waves in the interplanetary medium. The properties of shock waves mentioned by Kantrowitz show that even if the original outburst possessed no very sharp beginning, a sharp

front would be built up during the propagation through space. I know of no other theory that can reasonably give the extreme suddenness of this phenomenon.

LIEPMANN: I would ask whether the picture of a shock wave really is applicable. The mean free path in the residual gas between the sun and the earth appears to be 4 or 5 times the solar radius (as was stated by Schatzman); the thickness of the transition region in a shock wave front is of the order of say 4 free paths, which would make between 15 and 20 times the solar radius. Now the distance from the sun to the earth is 23.000 times the radius of the earth, which is about 200 times the sun's radius. If the phenomenon needs 24 hours to move through this distance, it would need about one or two hours before the transition region will have passed over us. Hence we would not explain the sharp front about which Gold has spoken on the basis of these figures alone. In order to get agreement with Gold's values the mean free path would have to be considerably shorter, i.e. by a ratio of about 100 or else the mechanism of interaction of the wave with the field of the earth has to explain the very sudden rise observed.

GOLD: In considering the interaction between the stream and the residual gas one must not restrict oneself to the collision cross section of neutral particles, but one has to consider the much stronger electromagnetic interactions that may occur between the two ionised gases.

MENZEL: The magnetic field of the earth will act as a kind of "bumper" with which the wave coming from the sun would collide, at a distance of roughly 5 times the radius of the earth.

BIERMANN: There is an important difference between the wave picture and the phenomena which accompany a magnetic storm (cf. the last remark by Gold). The atomic matter in the space between the sun and the earth is of a very small density, and the quantity of matter produced in a single outburst of the sun will often be large compared with the quantity of matter present. The relations valid for shock waves have to be applied therefore with due regard to this state of affairs. It should further be remarked that magnetic storms of the strength as observed on the earth will be able to drive away ionized cometary molecules and possibly even small dust particles.

SEATON: Perhaps we should be careful not to assume that shock waves are the only mechanism which can produce sharp edges. Auroral curtains in the earth's atmosphere are certainly not due to shock waves. The charged particles move in the plane of the curtain which is also the plane of the magnetic lines of force.

COWLING: Auroral forms like "curtains" are probably due to electrical discharges in the earth's upper atmosphere. Such discharges can occur there because of the conductivity which they produce, even though a similar phenomenon cannot occur in the interstellar gas.

CHAPTER 18

RADIATIVE AND COLLISIONAL EXCITATION

BY

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In ordinary gaseous nebulae, ultraviolet radiation from a hot star excites the emission of the gas. Electrons detached by photoelectric ionization excite by collision ions of heavy atoms to metastable levels; transitions to lower levels result in the emission of characteristic forbidden lines. The recombination of protons and electrons produces the Balmer lines. In the absence of ultraviolet radiation, dissipation of mechanical energy may heat the gas and will lead to the excitation of emission lines if the electron temperature becomes high enough. It should be noted that the forbidden lines are always excited by collisions and that consequently the relative intensities of the forbidden lines cannot furnish any criterion which permits to decide between the two excitation mechanisms.

The intensities of the Balmer lines relative to the forbidden lines depend on the type of excitation, but the interpretation requires knowledge of the state of ionization and of the chemical abundances. The Balmer decrement also depends on the excitation. For instance, while in a nebula transparent to the Lyman α line optical excitation leads to emission of the hydrogen lines only due to the cascading to lower levels after recombination, collisional excitation of hydrogen populates in addition the lower levels directly from the ground state. But similar conditions can arise with certain types of optical excitation. A clearcut decision between radiative and collisional excitation will rarely be possible. In most cases, the discussion will lead to the question which type of excitation requires the most plausible assumptions regarding density, temperature, ionization and chemical abundances.

The recent investigation of the collisional excitation of hydrogen by Chamberlain [2] opens the way to a quantitative discussion whether collisional excitation may be responsible for the emission in nebulae which apparently have no stellar source of excitation.

The best known example of such an object is the Cygnus loop. Oort[6, 7] has suggested that this nebulosity represents an expanding supernova shell slowed down by interaction with the interstellar medium and that the radiation of the shell is caused by collisional excitation. Recently, Chamberlain[3] has tried to decide whether radiative or collisional excitation prevails on the basis of measurements of the surface brightness of the nebulosity in Hα, the [O III] lines 4959/5007 and the [O II] lines λ 3726/29. He found that any of the following assumptions represent these observations with sufficient accuracy:

Collisional excitation:

(a) nebula optically thick in the Lyman lines with $T_e = 20000°$
(b) ,, ,, thin ,, ,, ,, ,, with $T_e = 10000°$

Radiative excitation:

transparent nebula excited by a star
with black body spectrum with T_e 10000°–20000°.

All three solutions seem acceptable and no decision seems possible.

Actually some of the data used in Chamberlain's discussion need revision. The best value of the radial velocity of expansion of the Cygnus loop, derived from a new discussion of Humason's unpublished data, is 45 km/sec; the usually quoted value of 75 km/sec is the maximum possible value. The corresponding distance, obtained by combining the velocity of expansion with the proper motion at the edge of the nebulosity as determined by Hubble, is 300 parsec. The emission per unit of volume thus becomes larger than the value derived by Chamberlain using a distance of 500 parsec. In addition, the diameters of the filaments were estimated too large; the sharpest filaments actually are unresolved even on photographs with the 100-inch telescope. An accurate determination of the volume emissivity seems impossible; the true volume emissivity may well be more than 100 times larger than the value adopted by Chamberlain. However, it does not seem likely that a repetition of his analysis with improved data could lead to a clearcut decision between radiative and collisional excitation.

A much better way of approach was opened when it was found that certain spectrograms obtained by Humason could be calibrated with the aid of step exposures of planetary nebulae obtained by K. Seyfert, using the same spectrograph and the same photographic emulsion. It was thus possible to determine true relative line intensities for the Cygnus loop.

The mean values from 8 spectrograms of different filaments are given in Table 1.

TABLE 1

RELATIVE INTENSITIES IN THE CYGNUS LOOP

Wave length	3727	3869	3967			4363		4959	5007
Identification	[O II]	[Ne III]	[Ne III] + Hε	Hδ	Hγ	[O III]	Hβ	[O III]	[O III]
Intensity	100	9.5	6	3	5	5	10	15	38

Since the value 1.89 of the ratio $I\,(3727)/I(4959 + 5007)$ found here agrees very well with Chamberlain's value 2.00, it seems permissible to combine Chamberlain's measure of Hα with the values in Table 1 for a discussion of the Balmer decrement. The measured value for Hα, however, is only an upper limit since it may contain a contribution from the [N II] lines λ 6548/84. *

For a comparison of the observed decrement in Table 2 with values

TABLE 2

THE BALMER DECREMENT

Line	Observed	Radiative exc. 40000°	Radiative exc. 160000°	Collisional exc. 40000°	Collisional exc. 160000°
Hα	<1.86	1.86	2.24	3.03	2.94
Hβ	1.00	1.00	1.00	1.00	1.00
Hγ	.5	.55	.52	.44	.49
Hδ	.3	.34	.31	.24	.25

computed for various types of excitation, the electron temperature T_e must be known. T_e may be determined from the intensity ratio $I\,(4959 + 5007)/I(4363)$. The value 10.6 of this ratio in the Cygnus loop is unusually low, but similar values are found in planetary nebulae, e.g. IC 4997. Since at high temperatures T_e depends very strongly on the value of the intensity ratio, it is not possible to deduce more than some kind of lower limit for T_e. If it is assumed that the intensity ratio is less than 13, T_e is found to be higher than 80000 °K, on the

* Observations made after the meeting show that the [N II] lines are indeed present, with local intensity fluctuations. On the average, they contribute perhaps 20% of the intensity. The Balmer decrement therefore is distinctly lower than the data in Table 2 indicate.

basis of the collision cross sections computed by Seaton [8]. The value 10.6 would give 200000 °K for T_e. Somewhat lower values are obtained if the electron density N_e is much higher than 10^4 cm^{-3}. Actually, values of N_e between 3×10^3 and 5×10^4 cm^{-3} are found within the wide range of conditions admitted by the uncertainty concerning the true value of the volume emissivity, the value of T_e and the type of excitation. Since at high temperatures the Balmer decrement does not depend strongly on the temperature, a precise value of T_e fortunately is not needed.

The observed Balmer decrement is given in Table 2, together with the theoretical values for radiative excitation (case A2 of Menzel and Baker[4]) and for collisional excitation in a nebula transparent in the Lyman lines for electron temperatures of 40000° and 160000 °K. These types of excitation give the lowest Balmer decrement for radiative and collisional excitation, respectively. Other types need therefore not to be considered.

The observed decrement clearly rules out the possibility of collisional excitation. The observed decrement seems to be even lower than the lowest of all theoretical decrements which occurs for case A2 at 40000°, but the discordance is not serious in view of the low accuracy of the observed intensities.

At first sight, the necessity to assume radiative excitation seems to present a serious difficulty since no suitable high-temperature star has been found near the center of the nebulosity. It may well be that this fact has been overemphasized. The possibility that the exciting star is the companion of a star of lower temperature and cannot be found for this reason seems to have been entirely forgotten. It is, for instance, practically impossible to prove that the 7th magnitude star of type B7 near the center of the nebula does not have a somewhat fainter companion of very much higher temperature. Such an assumption is not entirely artificial. Conditions of this kind are found in planetary nebulae, such as NGC 1514 and NGC 2346 with apparent central stars of spectral types A0 and A5 respectively. These stars cannot provide the excitation, and the assumption that the excitation is provided by hot companions is virtually unavoidable. The fact that a double star may exist in the center of a planetary nebula is demonstrated in NGC 246; in this case the hot star is brighter than the second component which is a G-type star, possibly a subdwarf.

A second example of an object where collisional excitation may be expected is the radio source in Cassiopeia [1]. No suitable exciting star is visible, but the occurrence of random velocities of thousands of kilometers suggests strongly that dissipation of mechanical energy may

provide the excitation. The nebulosity consists of two different kinds of small filaments: sharp filaments with relatively small radial velocities and more diffuse filaments with high radial velocities and large proper motions. The sharp filaments show only Hα and the [N II] lines. A steep Balmer decrement, indicated by the fact that Hβ cannot be observed, is consistent with collisional excitation. The diffuse filaments show a spectrum in which the lines of [O I] and the green lines of [O III] are equally strong. The [O II] lines are faint, and Hα is too faint to be observed. The quantitative interpretation of the spectrum meets a peculiar difficulty. Large changes of the diffuse filaments have been found by Baade after an interval of only two years. Unless the electron density is much higher than 10^4 cm^{-3}, the ionization cannot follow the rapid changes of density and must depend on the previous history of the filaments. This makes a definite interpretation of the spectrum virtually impossible. The observed spectrum can be interpreted [5] as due to collisional excitation with $T_e = 10000°$ or radiative excitation with $T_e = 15000°$ at an electron density of 10^4 cm^{-3}. In this case, collisional excitation is inherently more probable in view of the violent internal motions in this unusual object. But, even under such extreme conditions the spectrum does not reveal unambiguously the nature of the excitation.

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CHAPTER 19

DISCUSSION ON LUMINOUS EDGES AND COLLISIONAL EXCITATION

Chairman: Prof. G. TEMPLE

VAN DE HULST: Photographs of the luminous edges seen in diffuse nebulosities show a peculiar morphological feature. The dark clouds seem to be compressed to roundish shapes (droplets) or longish shapes (elephant's trunks), the borders of which are luminous towards the sides of the illuminating stars. Superficially, it seems as if these clouds have a kind of surface tension. A number of fine examples are found in M 16. I should like to make a suggestion how these forms may originate.

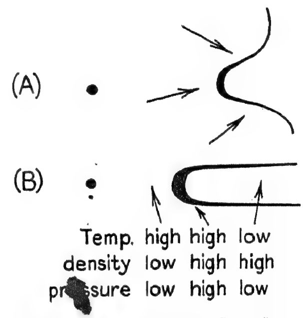

Fig. 1. Suggested development of elephant trunk.

Suppose an O star is born in a cloud complex with moderate but not excessive density fluctuations. The initial ionized region then has an irregular outline with somewhere a bulge (Fig. 1A). The pressure difference that is set up because the ionized region is hot and the unionized region is cool will make gas flow into the bulge. The density becomes higher in the bulge and lower in front of it. As a consequence the ionization is readjusted, giving a small ionized region in the denser gas. This may already be seen as a luminous edge. The tendency towards pressure equalization in the neutral and ionized gas will make more and more gas flow into the cold and dense bulge. The final situation may resemble Fig. 1B, in which we see a dark "elephant's trunk" with a marked luminous edge at the end that is directly illuminated by the star. Also the sides may be luminous because of ionizing radiation from the nebula.

These suggestions ignore entirely the more violent effect of a general expansion that is caused by the same tendency towards pressure equalization (See the discussion by Oort and others in Chs. 27–30). It is never-

theless possible that the peculiar forms may be formed in the way described during the expansion.

OORT: It is necessary to make a distinction between the luminous edges that may be bright borders of a dark cloud at the edge of a hot region and the thin wisps (of the Cygnus loop) that seem to be isolated in space. The stability of the latter wisps is still a problem to me. If Minkowski is satisfied that they may be excited by radiation, another question is still open: how are they kept together? Certainly, in the case of the Cygnus loop they have existed for a long time. The collisional mechanism in which an expanding shell pushes against an interstellar cloud gives a plausible explanation. A related question is whether the loops have been decelerated. Have they perhaps been expelled with a velocity of a few 1000 km/sec?

SAVEDOFF: How does one obtain the spectra of luminous edges?

MINKOWSKI: With high-aperture spectrographs and preferably not too big telescopes because the objects are fairly large.

VAN DE HULST: In one spectrum of the Cygnus loop taken at McDonald I found a very strong change of the ratio of $H\gamma$ to 4363 along the length of the slit. How should this be explained?

MINKOWSKI: This may be due to a local variation in electron temperature. Perhaps there is a hot spot.

LIEPMANN: Is it certain that the physical condition of the Cygnus wisps is stationary?

MINKOWSKI: Yes, the relaxation times are of the order of a few years, while the loop has been observed for more than 60 years.

ZANSTRA: The ratio of Balmer continuum to $H\beta$ gives a fairly reliable determination of the electron temperature for a number of planetary nebulae, though it should be used with some caution. I tried it out for Menzel and Aller's data (Ap. J. 93, 195, 1941), excluding NGC 7027, and got about 20,000° on the average, and I think a similar value might follow from Page's observations of $Ba_c/H\delta$ (Ap. J. 96, 78, 1942). At the time this value seemed too high on account of the determinations from the ratio $\lambda\, 4363/(N_1 + N_2)$, but they have recently gone up to about 20,000° average, so that it might be worth considering, as no doubt many are aware of. Can it be applied to the Cygnus loop?

MINKOWSKI: Unfortunately, the continuous spectrum is too weak.

KANTROWITZ: Coming back to the laboratory work, I should like to make two remarks. (a) The excitation by collision in shock waves is apparently quite different from collisional excitation processes which have previously been studied. (b) We found that a luminous contact

zone between hot argon and cold hydrogen gas emits Hα. Can this be explained?

MINKOWSKI: Unfortunately the excitation cross-sections between neutral atoms are very poorly known.

HOYLE: Two questions: is the total mass of the Cygnus loop known and have Lyman α quanta been taken into account?

OORT: The total mass may be about one tenth of that of the sun.

ZANSTRA: For a gaseous nebula the solution except for the population of level 2 is invariant under the inclusion or exclusion of the Lyman line emission by the gas itself, providing it is opaque at this frequency. For a nebula excited by stars of high temperature this is a well-known solution (case B of Menzel and Baker). The same invariance should hold for Chamberlain's treatment for collision excitation. (Indeed in his publication Ap. J. **117**, 387 (1953) Chamberlain in Tables 3 and 4 gives the values of b_n and the Balmer decrement for various electron temperatures in a nebula, optically thick in Lyman line radiation, which are independent of the optical thickness).

There followed a confused discussion between THOMAS, SEATON, ZANSTRA and others on the differences in the computations of collisional excitation.

SEATON: The point I originally made was this: When protons and electrons are overwhelmingly more abundant than hydrogen atoms, recombination to excited states will be much more important than collisional excitation. In the course of the discussion Oort replied that the radiative capture cross sections will be many orders of magnitude smaller than the cross sections for collisional excitation. Provided that the electron temperature is sufficiently high it is therefore quite possible that collisional excitation may be important. I am in complete agreement with Oort's reply.

THOMAS: It seems to me that the situation is being made much too complex. This problem is a straightforward one, and the situation including collisions has been treated by Giovanelli and myself, as well as Chamberlain, in various contexts, I do not believe the precise cross-sections used can introduce the discrepancy mentioned by Seaton, as suggested by Minkowski—I think it is simply the case that one writes down the equations and solves them, but cannot estimate energy-level populations from recombination coefficients alone, as Seaton is apparently doing. I tend to agree with Seaton in that I had not imagined collision processes to be particularly relevant in this planetary nebulae

problem ($N_e \backsim 10^4$, $T_e \backsim 10^4$), but the point should come out directly from the solution which Chamberlain has made.

I do not understand how Zanstra can make the statement that the solution is invariant under the inclusion or exclusion of the Lyman emission by the gas itself, providing it is opaque at this frequency. There must be a difference in the solution according or not as one includes the self-emission from the nebula, and whether this occurs in significant amount, i.e. whether or not the opacity is high. Certainly in the chromosphere case, which we have investigated in some detail, the population of the Balmer ground state depends by several orders of magnitude upon the inclusion of this Lyman region.

In regard to the existence of an electron temperature defined by a Maxwellian distribution for the electrons, one can show the electron temperature to exist and equal the atom kinetic temperature (to a very high degree of approximation) in the steady-state-case. In the region of a strong shock-wave, one can obtain a difference between electron and atom kinetic temperature, and presumably some non-trivial departure from a Maxwellian function.

Postscriptum by Zanstra: I think that the confusion between Thomas and myself arose largely from the fact that he has been working on the chromosphere, where physical conditions are quite different from those prevailing in gaseous nebulae. The reason is the extreme dilution of the radiation in a nebula, excited by a star, or the scarcity of collisions in Chamberlain's pure collision excitation of a nebula. In a planetary nebula a hydrogen atom remains unionized for several years. During this time it often happens that it is excited by Lyman line radiation, but the total time during which it is excited is small compared with its life in the ground state. It is true that the Lyman α radiation is bottled up, but yet it can escape sufficiently, the more since the scattering is non-coherent with complete re-distribution due to thermal Doppler effect, and it escapes in the line wings. It may be true that the solution changes if the $L\,\alpha$ optical depth would become enormous, so that there Thomas would be formally right, but under actual conditions in a nebula the $L\,\alpha$ optical depth, though large, is still small enough for this escape, and the approximation is a very good one.

CHAPTER 20

THE COLLISION OF TWO HIGHLY IONIZED CLOUDS

BY

F. D. KAHN
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There has been some talk at this colloquium about the effect of long mean free paths in a collision between two rarefied ionized clouds. It was assumed that a particle having kinetic energy U and charge ε will continue on its path without sensible deviation until it passes by another charged particle within a distance of the order ε^2/U, or less. This assumption leads to theoretical mean free paths of enormous length: for a stream of corpuscles expelled from the sun it exceeds an astronomical unit (cf. the discussion in Ch. 17). It is hard to understand the behaviour of such a stream on this basis.

To take some definite values, consider a collision between two clouds, each containing 10^2 protons and 10^2 electrons per cm^3, which are moving with speeds of 10^8 cm/sec in opposite directions. The thermal energy of the particles is taken to be much smaller than their kinetic energy. With the above assumptions the electrons of one cloud can penetrate a distance 3.10^{13} cm into the other cloud. For the protons the distance is 10^{20} cm.

But this situation is unstable. For suppose that the two clouds are counter-streaming in this manner. Then one may show, by a theory similar to that used for travelling wave tubes [1], that any small longitudinal oscillation among the electrons in the streams can be amplified indefinitely, provided its wave-number is sufficiently small. The critical value is

$$k_e = \left(\frac{8\pi\, N\varepsilon^2}{m\, V^2}\right)^{1/2},$$

where N = number of electrons per cm^3, in each stream,
V = speed of each stream,
m = electronic mass.

In the present example $k_e = 0.007$ cm^{-1}.

Since it is unlikely that the streams are absolutely uniform there will always be a series of small oscillations present, ready to be amplified. The relative energy of the electrons is therefore fed into plasma oscillations within a distance of the order of 10^3 cm, and their mean motion is stopped.

At first the protons continue their counter-streaming undisturbed but after a distance of the order of 10^5 cm interactions between them and the stationary electrons become important and oscillations are again amplified. The proton streams are therefore also brought to rest. The whole energy of the counter-streaming is therefore fed into space charge oscillations long before the individual particles of opposite streams have any effect on one another.

Such collisions may occur in sources of radio noise, like the one in Cassiopeia, where violent motions of ionized gases have been observed. The excitation of space charge oscillations probably has some bearing on the noise generation, for it is now known that purely thermal effects are not powerful enough.

It is hoped that a more detailed discussion will soon be published elsewhere.

REFERENCES

[1] R. Kompfner, Rep. Prog. Phys. 15, 316 (1952).

Discussion

Taylor: Is there a similarity solution of the equations developed by Kahn?

Hayes: There is no such solution. Since the approach velocity and ion densities at infinity are constant, the similarity expansion must be one in $\xi = x/t$. With this behaviour the acceleration of any ion species must vary as $1/t$. The same applies to the field strength, so that $E = (1/t) \cdot f_1(\xi)$. However, the relation between field and charge (Gauss' or Poisson's law) is such that the accumulated charge or field strength must vary as t, hence that $E = t \cdot f_2(\xi)$. These two forms of behaviour for E are inconsistent unless $E = 0$, which would give only the trivial unperturbed solution.

PART IV

TURBULENCE AND MAGNETIC FIELDS IN A COMPRESSIBLE GAS

CHAPTER 21

TURBULENCE AND MAGNETIC FIELDS

BY

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In a discussion before the Symposium, Lighthill and I arranged that he would talk mainly about specific effects of compressibility and I would confine my attention to problems involving the presence of a magnetic field.

Magneto-hydrodynamics is too large a subject for a survey to be attempted today, and in any case much has been written since the Paris meeting. I propose to mention briefly three different problems, in which turbulent kinetic energy and magnetic energy play important parts.

(a) Firstly I should like to remind people that the determination of the asymptotic level of magnetic energy in a medium of high conductivity which is in statistically steady homogeneous turbulent motion, is still an unsolved problem—or, at any rate, is still a disputed problem—and it would be useful to hear of any recent developments. The suggestion made by Biermann and Schlüter, Fermi, and Alfvén, is that the magnetic energy will increase until it is comparable with the kinetic energy (per unit mass of the medium), and I shall leave it to these people to restate their arguments during the discussions. My own view is that only the energy of relative motion of neighbouring particles (so close together that the lines of **H** are straight) is available for equipartition with the magnetic field, and we know from Kolmogoroff's theory of small-scale similarity that this amount of energy is of order $(\nu\,\varepsilon)^{\frac{1}{2}}$ per unit mass ($\nu =$ kinematic viscosity, $\varepsilon =$ rate of dissipation of kinetic energy per unit mass: $\varepsilon \backsim (\overline{u^2})^{3/2}/l$). The ratio of the two estimates is

$$\overline{u^2}/(\nu\,\varepsilon)^{1/2} \backsim \frac{l^{1/2}(\overline{u^2})^{1/4}}{\nu^{1/2}} = (\mathrm{Re})^{1/2}$$

(l = length scale of turbulence, $\overline{u^2}$ = mean square turbulent velocity), which may be quite large.

(b) Secondly, I have agreed to report on a short paper by Chandrasekhar and Fermi (Astrophys. Journ. **118**, p. 113, 1953), who have been unable to come to the Symposium in person. This paper describes two independent methods of estimating the magnetic field in the spiral arm containing the solar system, and both methods have possible application to other problems. The first method makes use of the inference (which may itself be questioned), from the observed polarization of the light from distant stars, that the magnetic field in the spiral arm is approximately parallel to it, with a mean angular deviation of $\alpha = 0.2$ radians. The observed random variations in direction are regarded as being due to the turbulent motion within the spiral arm dragging the magnetic lines of force to and fro. The lines of **H** act as stretched elastic strings, and the angular deviation of the line is determined by the ratio of the imposed lateral velocity due to the turbulence (u_t, say) to the phase velocity of waves travelling along the lines, i.e.:

$$\alpha \backsim \frac{u_t}{\text{velocity of magneto-hydrodynamic waves}} = \frac{(4\pi\varrho)^{1/2}\, u_t}{H};$$

Then from the data assumed by the authors ($\alpha = 0.2$, $\varrho = 2 \times 10^{-24}$ gm/cm^3, $u_t = 5 \times 10^5$ cm/sec) H is found to be 7.5×10^{-6} gauss.

The second method assumes that the spiral arm is in equilibrium, so far as lateral motion is concerned, so that the kinetic and magnetic pressures together are balanced by the gravitational tension. The magnetic pressure is $H^2/8\,\pi$, the kinetic pressure is of order $\varrho\, u^2$, where u is the r.m.s. particle velocity or turbulent velocity whichever is the greater, and with the assumption of a cylindrical spiral arm of radius R the gravitational tension is $\pi\, G\, \varrho^2\, R^2$. The balance of forces then gives a value of H of order 6×10^{-6} gauss.

The close numerical agreement between these estimates is perhaps fortuitous, but the methods of estimation are interesting.

(c) My third remark concerns a problem which may play a part in the understanding of interstellar gas clouds, *viz.* what happens to the turbulent kinetic and magnetic energies when the medium is expanded or contracted? In its general form this problem is very difficult, but it becomes tractable in the case of certain simple kinds of distortion. The simplest case is that in which the medium in turbulent motion, and with a given magnetic field, is subjected to a very rapid uniform isotropic strain. If the time scale of the straining motion is small enough, all

regions of the fluid will undergo the same strain, and the lines of vorticity and magnetic force will follow the fluid in its motion. Since the strain is isotropic, distributions of vorticity and of magnetic force are unchanged (when referred to strained axes) so far as direction is concerned, but their magnitudes everywhere change by a factor e^{-2} (where e is the extension ratio of the strain) in order to preserve the fluxes. Hence the velocity u is proportional by to e^{-1}, whereas $H \backsim e^{-2}$, and we have the following results:

$$\text{kinetic energy per unit volume} = \tfrac{1}{2}\,\varrho\,\overline{u^2} \backsim e^{-5}$$
$$\text{magnetic ,, ,, ,, ,,} = (\mu/8\pi)\,\overline{H^2} \backsim e^{-4}.$$

It follows that a rapid expansion of the meduim will have the effect of diminishing both kinds of energy, and will diminish kinetic energy more rapidly than magnetic energy. A rapid compression will build up both energies, particularly the kinetic energy. These results can also be written in the form

$$\text{turbulent kinetic pressure} \backsim \varrho^{5/3},$$
$$\text{magnetic pressure} \backsim \varrho^{4/3},$$

the 5/3 appearing for the simple reason that a continuous fluid medium has only the three translational degrees of freedom.

A few other simple cases can be considered. If a flattened rotationally symmetrical disk of gas is subjected to a rapid further contraction in the direction of the axis of symmetry, it is found that the turbulent kinetic pressure opposing the contraction is proportional to ϱ^3, although there is no corresponding simple result about the magnetic pressure. It is also possible to find results for a distortion which is so slow that the turbulent motion is in equilibrium with whatever is feeding it with energy (e.g. a differential rotation) at each stage of the distortion.

The implication of these results is that whenever the gas kinetic pressure is not dominant (and it will not be if the Mach number of the turbulence is of order unity or more), it will be necessary to make use of special "equations of state" for the turbulent and magnetic pressures. These dynamical "equations of state" may be rather complicated and will depend on the way in which the change in density of the gas is brought about, i.e. on the speed of the change and on the directional characteristics of the distortion.

Discussion

E. C. Bullard: The possibility of dynamo action is important for geophysics as well as astrophysics. Batchelor has shown that turbulent motions in a conducting fluid may produce a magnetic field. Turbulent motions provide a relatively inefficient method of producing a field, and it is to be expected that there are large-scale motions that will do so with smaller velocities than Batchelor requires.

A detailed investigation has been made of a velocity system that seems likely to bear some resemblance to that which might occur in the core of the earth, where Batchelor's mechanism will not work. A paper describing this has been submitted for publication in the Phil. Trans. of the Royal Society.[1] In detail the results are of more interest for terrestrial magnetism than for interstellar problems, but there may well be circumstances where these calculations provide a guide to what may happen and to possible methods of calculation. The motions considered consist of a combination of a cellular convection with a rotation which varies with the radius.

[1] Phil. Trans. Roy. Soc. A. 247, 213 (1954).

CHAPTER 22

THE EFFECT OF COMPRESSIBILITY ON TURBULENCE

BY

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1. INTRODUCTION

If any applications of turbulence theory to astrophysics are to be made it will be necessary to consider what effect compressibility may have on the postulated motions. This paper describes what is known, or can reasonably be conjectured, about the influence of compressibility on the turbulent motion of fluids, but makes no attempt to apply the results to any astrophysical problem.

The principal fluid-dynamic effect of a finite stiffness in compression is that readjustments in pressure distribution are propagated with a finite speed instead of instantaneously. Radiated sound results, since (crudely speaking) the *required* readjustment of pressure has changed by the time the signal arrives; the difference is radiated as sound. Thus the chief influence of compressibility on a phenomenon like turbulence will be that it is constantly radiating energy in the form of sound. This sound energy must ultimately be converted into heat by the various processes of acoustic attenuation, which probably take place nearly independently of any scatterings of the sound due to the turbulence. Accordingly, compressibility acts as a source of energy dissipation, additional to that provided by shear viscosity.

A detailed quantitative theory of the process, with good experimental backing, [1, 2, 3] exists only when the root mean square Mach number of the turbulence is small compared with 1. For larger values of the root mean square Mach number, one can make only tentative conjectures, as will be done at the end of this paper (§ 4). According to these conjectures, the influence of compressibility becomes dominant for root mean square Mach numbers comparable with 1, or greater.

In the account of the theory (§ 2), the experimental evidence will be omitted, for the following reasons. It has been found possible so far to

measure the acoustic output of turbulence only in the case of free jets. In these experiments, the acoustic output was sufficient to make reliable measurement possible only when the Mach number of the stream emerging from the jet orifice exceeded 0.3. Agreement with theory was then obtainable only after the theory had been modified to take into account the convection of the turbulence at a non-negligible Mach number through the atmosphere into which it radiates. The experimental evidence cannot be appreciated without this modification to the theory, which however there is not time to describe in the present paper. The reader is accordingly referred to a paper by the author [3] for the detailed comparison of theory with experiment.

2. THE SOUND GENERATED BY TURBULENCE WITH SMALL ROOT MEAN SQUARE MACH NUMBER

Turbulence does not generate sound like a siren; the fluctuations of pressure at a point, with their corresponding fluctuations of density, are unable to produce an acoustic source field in the manner of the puffs of steam in that notoriously efficient instrument. This is because in the turbulence no new fluid is introduced; hence the total source strength is zero and the effects of the different hypothetical sources would nearly cancel out at large distances.

Turbulence does not even generate sound in the somewhat less efficient manner of, say, a bell; the fluctuating force with which one eddy acts on the rest of the fluid is unable to produce a dipole field like that produced by the "puffs of momentum" given to the air by the bell surface. This is because no external force acts on the turbulent fluid; the internal actions and reactions balance; hence the total dipole strength is zero, and the effects of different hypothetical dipoles nearly cancel out at large distances.

Actually, the sound generated by turbulence is an effect of the next order still, and is basically a quadrupole field generated by fluctuations, not of mass (say ϱ per unit volume) or momentum (say ϱv_i per unit volume in the x_i-direction, for $i = 1, 2, 3$) but of the *momentum flux*, which is $\varrho v_i v_j$ per unit volume for flow of x_i-momentum in the x_j-direction. As a tensor it includes the kinetic energy $\frac{1}{2} \varrho v_i^2$, which is half its "trace" (sum of diagonal elements). It includes also *lateral* momentum flux, i.e. flow of momentum perpendicular to itself, and we shall see that it is this which is responsible for the bulk of the sound generated.

To see mathematically why the sound field of turbulence is that of a

distribution of quadrupoles of strength $\varrho\, v_i v_j$ per unit volume, one may write the equations of continuity and momentum in the form

$$\frac{\partial \varrho}{\partial t} + \frac{\partial(\varrho v_i)}{\partial x_i} = 0, \quad \frac{\partial(\varrho v_i)}{\partial t} + \frac{\partial(\varrho v_i v_j)}{\partial x_j} + a^2 \frac{\partial \varrho}{\partial x_i} = 0. \tag{1}$$

Here a is the velocity of sound; the replacement of pressure gradient by a^2 times the density gradient, and the neglect of the viscous stresses, can be justified by a detailed discussion of the magnitude of the sound field of the terms neglected [1,2].

Eq. (1) would yield the ordinary equation of sound if the term due to momentum flux $\varrho\, v_i v_j$ were absent. This term, neglected in ordinary acoustic theory, generates a sound field in the way that externally applied stresses would do. This field is a quadrupole field, essentially because a stress field implies equal and opposite forces on each side of a fluid element, and a force is acoustically equivalent to a dipole. As an alternative argument, eliminating $\varrho\, v_i$ from equations (1), we have

$$\frac{\partial^2 \varrho}{\partial t^2} - a^2 \nabla^2 \varrho = \frac{\partial^2}{\partial x_i \partial x_j} (\varrho\, v_i v_j), \tag{2}$$

and the double derivative in the "forcing term" on the right shows that it represents a quadrupole field of strength $\varrho\, v_i v_j$ per unit volume.

The retarded potential solution of (2) is

$$\varrho - \varrho_0 = \frac{1}{4\pi a^2} \frac{\partial^2}{\partial x_i \partial x_j} \int \frac{1}{r} [\varrho\, v_i v_j]\, \partial\tau, \tag{3}$$

where r is the distance of the point x_i from the element of integration and the brackets signify the value at the "retarded" time $t - r/a$. At large distance from the turbulence (in the "radiation field") this becomes

$$\varrho - \varrho_0 = \frac{1}{4\pi a^4} \int \frac{r_i r_j}{r^3} \left[\frac{\partial^2(\varrho v_i v_j)}{\partial t^2}\right] \partial\tau, \tag{4}$$

because it is only through the variation of the retarded time with x_i that any sound with amplitude falling off like r^{-1} is present in (3).

The strong dependence of (4) on frequency is typical of multipole radiation. Ordinary dipole radiation is already inefficient at low frequencies, for just this reason—or, physically, because an equal source and sink cancel out in their effect at a large distance, except in so far as the retarded time for each is slightly different, and the difference of strengths at the different retarded times is small if the frequency is low. The argument applies twice as effectively for quadrupoles.

The fact that, by (4), the amplitude of the sound produced varies as the square of the turbulent velocities multiplied by the square of their

frequency, means that, for a given length scale l of turbulence, it will vary as the fourth power of the root mean square velocity u. (For a typical frequency is proportional to u/l.) Hence the *energy* radiated should vary as u^8.

We can get a slightly more precise result as follows. The acoustic intensity is a^3/ϱ times the mean square of the amplitude (4). This involves the correlation between the values of $\partial^2(\varrho\, v_i v_j)/\partial t^2$ at different points, which will be significant only when the points are fairly close—sufficiently so, in fact, for the difference between the retarded times for each to be negligible. It follows that the intensity field *per unit volume* of turbulence can be put into the form

$$\frac{\varrho}{16\pi^2 a^5}\,\frac{r_i r_j r_k r_l}{r^6}\int \overline{\frac{\partial^2(v_i v_j)}{\partial t^2}\,\frac{\partial^2(v_k' v_l')}{\partial t^2}}\,\partial\tau', \tag{5}$$

where the bar signifies a mean. Since the integral over the dashed points extends only over points sufficiently near to the undashed point for the mean product to be non-negligible, it is roughly proportional to an *average eddy volume*, or to l^3. This shows that the sound energy radiated per unit volume of turbulence is proportional to $\varrho u^8/a^5 l$.

Comparing this with the rate of viscous dissipation of energy, which is fairly closely $\varrho u^3/l$, we see that the acoustic efficiency of turbulence is proportional to the *fifth power of the root mean square Mach number.** The very low efficiency which this implies at low Mach numbers is a direct consequence of the sensitivity to frequency which results from the quadrupole character of the field.

Proudman[3] has made an approximate calculation of the integral in (5) from isotropic turbulence theory, and obtains a uniform directional distribution of intensity with a total energy output

$$E = 40\,\frac{\varrho u^8}{a^5 l} \tag{6}$$

per unit volume. The acoustic efficiency is therefore

$$40\left(\frac{u}{a}\right)^5. \tag{7}$$

It is interesting that the coefficient in (6) and (7) is of a higher order of magnitude than 1; if this happens when a calculation succeeds a dimensional analysis, one should try to determine the physical reason

* Here we assume a steady level of turbulence, so that the energy input can be taken equal to the dissipation rate.

for it. (One may remark that the factor of order 40 is almost indispensable in getting agreement with the experimental data [2].)

To find this reason, we return to the conservation of mass and momentum which led to the absence of source and dipole terms, and the non-conservation of the momentum flux, which is responsible for the quadrupole field $\varrho\, v_i v_j$ not degenerating into mere octupoles. Mathematically, a quantity satisfies a conservation law if its time rate of change can be written as a *divergence*, so that its rate of change in any volume is due solely to inflow across the boundary (note that any divergence on the right of equation (2) increases by one the order of the multipoles which are present). Now when one tries to express $\partial(\varrho v_i v_j)/\partial t$ as a divergence one finds that one term is left over, namely

$$p\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)=p e_{ij}, \tag{8}$$

where p is the local excess pressure and e_{ij} is the rate-of-strain tensor. The "trace" of the tensor (8) is small (zero for "incompressible" flow); one may conclude that little sound results from fluctuations in the kinetic energy ($\frac{1}{2}\varrho v_i^2$ per unit volume). But the rate of change of *lateral* momentum flux contains a term equal to the *product of the excess pressure and the rate of shear* (the other terms, as stated, producing weaker sound fields). Hence it is the combination of excess pressure and shear at a point, in such a way that their product varies, which constitutes the basic source of generation of noise by turbulence. The mechanism by which it does so is indicated graphically in Fig. 1.

The sound generated by turbulence consists, therefore, of an aggregate of lateral quadrupole fields of the kind illustrated in Fig. 1. For isotropic turbulence all quadrupole orientations are present in equal proportions, and so the observed field is non-directional. For heavily sheared turbulent flow, however, the mean shear gives the quadrupoles a predominant orientation, which is observed.

For isotropic turbulence, the importance of the analysis just sketched lies in the fact that the terms like $\partial^2(v_i v_j)/\partial t^2$ in (5) may be replaced by single time-derivatives of the product of pressure and rate of shear. In these it is only the rate of shear which is likely to exceed markedly its estimate (u/l) given by dimensional considerations alone. Actually for large Reynolds numbers $u\, l/\nu$, the *root mean square* rate of shear exceeds u/l by a very large factor, of the order of the square root of the Reynolds number. However, shears as large as this cannot be important, owing to the very small "average eddy volume" (see above, following

equation (5)) which is associated with them. A detailed scrutiny of Proudman's calculations shows [2, 3] that it is velocity gradients of the order of $8u/l$ (or, more accurately, whose time rate of change is of order $8u^2/l^2$) which are chiefly responsible for the sound generated, and that this factor 8 (which must appear *squared* in the intensity) explains the factor of order 40 which appears in the expression (6) for total energy output. It seems to be an intermediate size of eddy, between the main energy-containing eddies and those which dissipate the energy, which generates most sound.

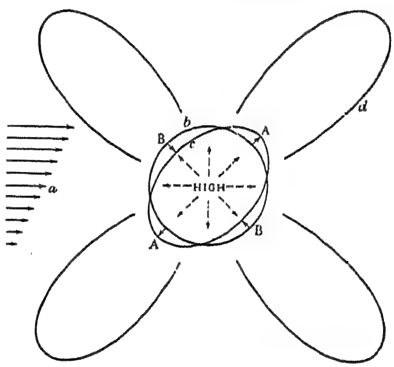

Fig. 1. Basic lateral quadrupole resulting from combined excess-pressure and shear at a point. Shear (a) in small time-interval deforms spherical fluid element (b) into ellipsoid (c) with principal axes at 45° to the flow. Excess pressure (HIGH) causes momentum outwards from it to increase (brokenline arrows). At A momentum is being created in the direction of deformation, so there is *increasing* momentum flux outwards. At B momentum is being created in the direction opposite to that of deformation, so there is *decreasing* momentum flux outwards. Fluctuations in this pattern of changing momentum flux, due to fluctuations of either pressure or shear, produce sound radiation with the polar intensity diagram (d).

3. SOME INFERENCES FROM THE THEORY

Having determined that the radiation field of unit volume of isotropic turbulence is directionally uniform, with total power output E given by eq. (6), one may go on to determine the pressure fluctuations at any point within the turbulent fluid. The pressure fluctuations needed on incompressible flow theory to balance the *local* eddy motions were calculated by Batchelor [4]; their mean square is

$$\overline{p^2} = 0.34\,\varrho^2 u^4. \tag{9}$$

To these must be added the fluctuations in pressure due to the sound field of all the rest of the turbulence. Now for an infinite volume of turbulence, and sound obeying an inverse square law, the sound arriving at any point would be infinite, since the volume of turbulence at a distance between r and $r + dr$ from the point would be proportional to $r^2\,dr$, while the sound generated in this volume would be reduced by a factor of only r^{-2} before reaching the point. In a real fluid, however, the sound energy would be attenuated by an additional factor $e^{-\alpha r}$, where α is the rate of dissipation of unit acoustic energy per unit

distance, depending on viscosity, heat conduction and other processes of energy exchange between molecules or between various degrees of freedom of a single molecule. In this case it is easy to see that the intensity at the point will be $a^{-1}E$. (In practice, a varies with frequency, so the value of a^{-1} here would have to be taken as a mean weighted with respect to the acoustic energy spectrum.)

Hence the mean square pressure due to acoustic fluctuations at the point is $\varrho a a^{-1}E$, and since these fluctuations must be uncorrelated with the pressure fluctuations due to the local motion, whose mean square is given by (9), the two may be added to give

$$\overline{p^2} = \varrho^2 u^4 \left(0.34 + 40\,\frac{u^4}{a^4}\,\frac{1}{al}\right). \tag{10}$$

Evidently, the acoustic part of the pressure fluctuations may begin to be dominant at quite moderate root mean square Mach numbers, especially if the acoustic attenuation in a distance equal to the length scale of the turbulence is small*.

Similarly one can write down the mean square velocity at a point as a sum of terms due to the fluctuating shearing motions and the fluctuating longitudinal motions (sound waves), namely

$$\overline{u^2} = u^2 \left(1 + 40\,\frac{u^6}{a^6}\,\frac{1}{al}\right). \tag{11}$$

In this expression, the Mach number at which the acoustic component would begin to predominate would be somewhat larger.

4. CONJECTURES CONCERNING TURBULENCE AT ROOT MEAN SQUARE MACH NUMBERS COMPARABLE WITH 1, OR GREATER

It has been shown that the effect of compressibility is to increase the rate of turbulent energy dissipation by a factor of about

$$1 + 40\,M^5, \tag{12}$$

where $M = u/a$ is the root mean square Mach number, assumed small. This formula cannot reasonably be applied when M is greater than about 0.5, since the hypotheses on which it is based (particularly the neglect of the difference in the retarded times for the two terms of the mean product in (5)) are then invalid. However, it can hardly be doubted that the factor by which the dissipation is increased is large when M is

* This proviso is likely to be true, since under the conditions envisaged the length scale l will be less than a typical wavelength of the sound generated.

comparable with 1, or greater. Accordingly, the power input of the turbulence-creating mechanism (by which turbulent energy is extracted from the kinetic energy of some mean shearing motion, or from the potential energy of some unstable distribution of fluid in a field of force, or otherwise) must be far greater, relative to the level of turbulent energy which is generated, than it would need to be at low Mach numbers*.

It is clear, also, that the sound amplitudes present for Mach numbers M comparable with 1 will be very great. It is reasonable, therefore, to seek guidance on what may happen from the theory of sound waves of large amplitude. This is well developed only for the case of plane waves. These have the well-known property that their fronts rapidly steepen and become shock waves, while the rear part (expansion phase) of each wave becomes less and less steep [7]. Thus they become what are often referred to as "N-waves", from their characteristic shape (Fig. 2). A general (non-periodic) plane wave becomes a certain statistical assemblage, or random sequence, of N-waves. The degradation of energy in this case occurs almost entirely inside the shock waves, following Rankine's well known formula for the entropy increase per unit mass of fluid traversed by a shock wave of pressure ratio x, namely [7]

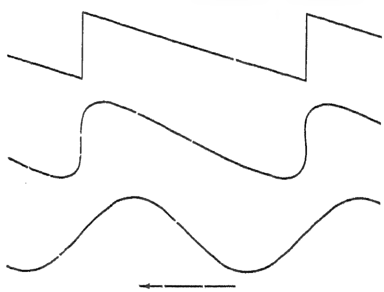

Fig. 2. Development of a sound wave of large amplitude into an N-wave. The curves shown are pressure-distance curves at successive times; time increases upwards and the wave is propagated to the left. The discontinuities formed, in which the further steepening of the wave is resisted by molecular diffusion, are "shock waves".

$$\Delta S = c_v \left[\log x - \gamma \log \frac{(\gamma + 1)\, x + \gamma - 1}{(\gamma - 1)\, x + \gamma + 1}\right], \tag{13}$$

where $\gamma = c_p/c_v$ is the ratio of the specific heats. Thus the presence of the shock waves speeds up very considerably the attenuation of the acoustic energy. Another prominent feature of such an assemblage of N-waves would be the "union" of shock waves whenever a stronger one catches up with a weaker one.

It may be remarked that just such a statistical assemblage of N-waves incorporating energy loss at the discontinuities and similar possibilities

* Similar considerations hold for shear flow turbulence [2], and are probably the cause of the reduced level of turbulence in jets at high Mach numbers.

of "union", was shown by Burgers[5] to occur in a certain highly simplified one-dimensional "model" of turbulence. In this model, the discontinuities were supposed to correspond to vortex sheets in the three-dimensional case, and indeed Batchelor and Townsend [6] have shown experimentally that such vortex sheets do appear to dominate the smallest-scale features of ordinary low-speed homogeneous turbulence. However the considerations just given indicate that Dr. Burgers's model (which does not use any equation of continuity, "incompressible" or otherwise) may give also some idea of the character of "compressible" turbulent flow at the higher Mach numbers, with the discontinuities corresponding to shock waves.

Extending the picture to three dimensions, one may imagine the turbulence to consist not only of the usual vortex motions, but also of a three-dimensional "statistical assemblage of N-waves"; that is, of shock waves of all shapes rushing about in all directions, with regions of more gradual expansion between them, and with continual interactions taking place between pairs of shock waves (including unions, regular intersections, "Mach" intersections [7]) and to a lesser extent between them and the longitudinal expansion waves and the shear turbulence [8]. The interactions between shock waves actually create additional vorticity; also, a single shock wave along which the entropy increase (**13**) is non-uniform creates vorticity, in proportion to the gradient of that increase [7]. Thus to some extent the shock wave system can generate new turbulence. Whether as a result of all this any kind of equipartition between the energy of longitudinal and shearing motions is likely to be set up can only be a matter of opinion. The author feels rather that the system has become one in which the division of the motion into "turbulence" on the one hand and "sound" (or shock waves) on the other is almost without significance.

In either case our main conclusion, after this enumeration of the classes of phenomena which are likely to play a prominent part, can only be that the properties of turbulence at root mean square Mach numbers comparable with 1, or greater, must differ very widely from those found in incompressible flow.

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CHAPTER 23

INTERNAL MOTION OBSERVED IN EMISSION REGIONS

BY

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(Communicated by E. SCHATZMAN)

Since 1949 Strömgren and Fehrenbach [1] have explored the Milky Way in Hα light by means of high speed photographic cameras with interference filters. Numerous new emission regions have been found because of the better contrast between these regions and the air glow [2, 3, 4].

The intensity of this emission justified an attempt at measuring precise physical quantities by means of interferometer observations. The new Pérot-Fabry étalons with multiple layers give a considerable dispersion and contrast with a convenient speed [5]. The study of the air glow doublet 6300–6363 A by Cabannes and Dufay has shown the advantages of this method. With this same étalon I have obtained rings of the Hα-radiation in the entire constellation Cygnus in 3 hours on 103 *a* E plates.

The earlier studies [6, 7] have used the old étalon with half-silvered layers on the Orion nebulae, which was the only nebula bright enough for this study. The new étalon permits us to study even the faintest regions, as shown by the following report on the study of the nebulosity around λ Ori. The étalon is mounted in a parallel beam in front of a camera with focal ratio F/1,4. The same mounting is also intended to photograph nebulae through interference filters with big telescopes.

The measurement of a photograph of Pérot-Fabry rings gives the radial velocity at many points in the field so that turbulence can be studied very easily. The mean residual deviation of the measurements from the average velocity was found to be 1.7 to 3.3 km/sec in ten nebulae studied. The large nebula in Orion gave the higher value 5.8 km/sec.

The order of magnitude of the highest radial velocities is 10 or 15 km/sec in all of these nebulae. Even higher values are noted in some points; they are visible at first sight by the deformation or widening of the rings when the path difference exceeds 4000 μ. They seem to be

localized at particular morphological details (filaments, ends of bright arcs, etcetera). Also the regions bordering dark nebulae are often disturbed. A rough estimate of the temperature gives a value below 10.000° K.

The large Orion nebula and the faint nebulosity around λ Ori may be

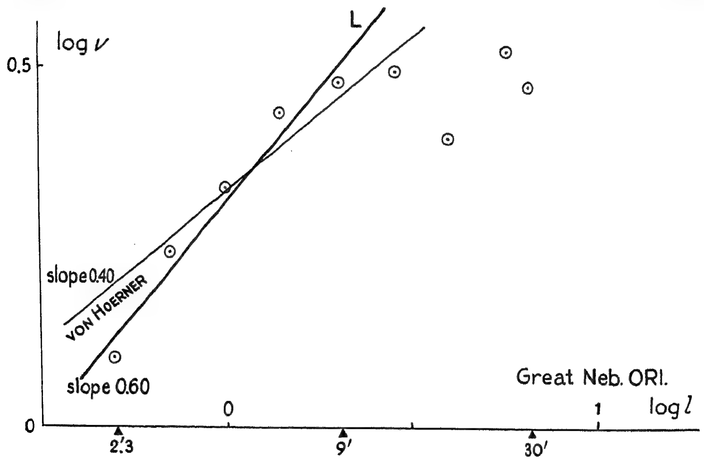

Fig. 1. Spectrum of turbulence of Orion Nebula.

considered as extreme cases. I have tried to check whether these nebulae obey Kolmogoroff's law as it has been formulated, for one velocity component, by von Weizsäcker: "In a homogeneous medium the relative velocity v between two points at a distance l is on the average proportional to $l^{1/3}$."

The results are shown in Fig. 1 and 2. Fig. 1 gives for the Orion nebula the values of log v (in arbitrary units) as a function of log l. The slope in the first part ($l < L$) is 0.60, where von Hoerner [8] found 0.40 (thin line) from Campbell and Moore's measurements [9]. My field extends more than 20′ from the trapezium but does not include the very brightest part. The fact that the slope is steeper than 1/3 is explained by von Weizsäcker by compressibility and by von Hoerner as due to an effect of optical depth. The difference in slope between von Hoerner's result and mine may then be due to the fact [10] that the optical depth is larger in H α (used by me) than in the [O III] lines (used by Campbell and Moore).

Fig. 2 shows the results for the nebula surrounding λ Ori, a weak and fairly regular nebulosity with locally some velocities exceeding 15 km/sec. The measurements extend to 150′ from λ Ori. The part of the graph with $l < L = 46'$ corresponds to Kolmogoroff's law.

The transition at $l = L$ in both figures may correspond to von Karman's transitional region [11] in the turbulent spectrum.

With the known distances [12], the linear values for L, i.e. the sizes of

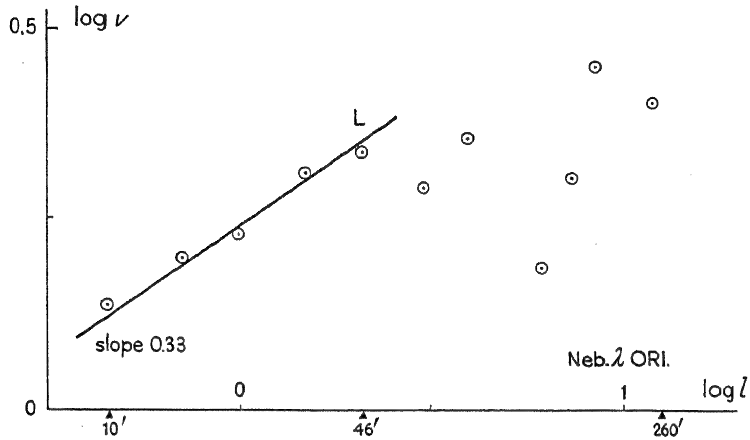

Fig. 2. Spectrum of turbulence of nebula near λ Orionis.

the turbulent elements, are 1.0 parsec and 8.0 parsec for Fig. 1 and 2, respectively. The latter value corresponds to the classical dimension of a H II region and also with the order of magnitude found by Aller [13] from the intensity fluctuations of emission nebulosities in Cygnus.

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CHAPTER 24

DISCUSSION ON THE MEASUREMENT OF TURBULENCE IN NEBULAE

Chairman: Dr. TH. VON KÁRMÁN

MINKOWSKI: Courtès has made the suggestion that the different slopes of the turbulent spectrum in different nebulae might be due to effects of optical depth. I am doubtful whether it might not play a role that in different nebulae the turbulent spectrum is affected to different degrees by the occurrence of small clouds with large motions. I have seen many plates of the Orion nebula showing such clouds with velocities perhaps up to 50 km/sec, but NGC 6523 (M 8), for instance, is much more quiescent.

LIEPMANN: On what kind of average are the plotted points based?

SCHATZMAN: Courtès reports that he takes many pairs of points PQ on one plate, all with the same mutual distance. He then determines the mean of the differences between the velocities $v(P)$ and $v(Q)$ and does the same for the other distances.

FRENKIEL: The interpretation of the results obtained by von Hoerner and by Courtès must be very uncertain. Each separate observation gives a mean value of the velocities over a certain depth in the line of sight and consequently is an average over various elements of volume, which may move more or less independently from each other, while the weight of the contributions is influenced by absorbtion. One now takes the differences between the values so obtained for settings on different points of the image of the nebula as projected on the celestial sphere. With a total number of say 50 or 60 observational points, one may perhaps obtain some general idea concerning the shape of the correlation curve or of the spectrum of turbulence, but I doubt if it determines them sufficiently well for a verification of Kolmogoroff's law.

KAMPÉ DE FÉRIET: In this method one observes from O, in a cloud C, the radial velocities along a given radius Ox; but the result does not depend only on the radial velocity $U(a)$ at the point A on the front of

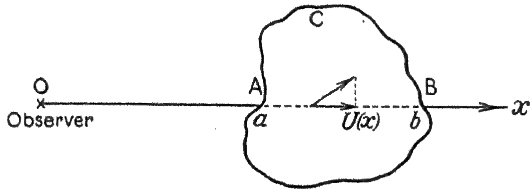

Fig. 1

the cloud; it certainly depends on the values $U(x)$ of the radial velocity at every point x in the cloud between A and B; what one is measuring is very likely the integral:

$$I = \int_a^b U(x)\, dx. \tag{1}$$

If the cloud is in turbulent motion let us write:

$$U(x) = \overline{U(x)} + U'(x),$$

where $U'(x)$ is a random function of x; it seems reasonable to assume that $U'(x)$ is a *Gaussian stationary random function*, i.e.:

(a) for every set of points $\{x_1, \ldots\ldots, x_n\}$ the n random variables $U'(x_1), \ldots\ldots, U'(x_n)$ have an n-variate Gaussian distribution;

(b) $\overline{U'(x)} = 0$;

(c) $\overline{U'(x)\, U'(y)} = \varrho\,(x - y)$.

Then:

$$I = \int_a^b \overline{U(x)}\, dx + J; \qquad J = \int_a^b U'(x)\, dx.$$

It is easy to prove that:

J is a random variable with a Gaussian distribution:

$$\text{Prob}\,[\xi < J < \xi + d\,\xi] = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{\xi^2}{2\sigma^2}}\, d\,\xi,$$

where:

$$\sigma^2 = \int_0^{b-a} (b - a - h)\, \varrho\,(h)\, dh.$$

If one feels the need (for instance in order to take account of the absorbtion of light in the cloud) to introduce a weight for the influence of $U(x)$ on the measurement, i.e. to replace (1) by:

$$I_1 = \int_a^b p\,(x - a)\,U\,(x)\,dx, \tag{2}$$

then

$$J_1 = \int_a^b p\,(x - a)\,U'\,(x)\,dx$$

is still a Gaussian random variable for which:

$$\sigma^2 = \int_o^{b-a}\int_o^{b-a} p(x)\,p(y)\,\varrho\,(x - y)\,dx\,dy.$$

This result seems to be useful if one wants to improve the method of Mr. Courtès, taking account of the depth of the cloud.

CHAPTER 25

REMARKS CONCERNING INTERSTELLAR MAGNETIC FIELDS

BY

L. BIERMANN
Göttingen

Concerning the remark made by Bullard I wish to point out that an increase of the magnetic field strength is possible if $\sigma\nu/c^2$ exceeds unity, where σ is measured in E.S. Units, and $\sigma\nu/c^2$ amounts to

$$\sigma\nu/c^2 \lesssim 10^9 \quad \text{(in H I-regions)}$$

$$\text{or} \qquad 10^{10} \quad \text{(in H II-regions).}$$

This is amply sufficient. The value of σ to be used in this connection, is the "normal" one, that is, it is not affected by the magnetic field (see Schlüters contribution and the consequent discussion on Tuesday; this was not yet known, however, at the Paris meeting).

Every irregular motion tends to give rise to a different acceleration to electrons and ions; or otherwise stated to such motions always "impressed electric forces" correspond. If e.g. there would be no interstellar magnetic field at all, the expansion of one particular H II cloud would create a magnetic field of the order 10^{-17} Gauss. The theory of the origin of magnetic fields by the action of such forces has been worked out to some extent [1]. Experiments relating to this mechanism have been made in Kiel; they may to some extent be regarded as models for the processes under discussion [2].

On the other hand, the possibility cannot be excluded, that interstellar magnetic fields were present already in the earliest stages of our galaxy.

I turn now to the dynamical state of the interstellar gas and its connection with large-scale magnetic fields. The two opposite positions, which have been defended in the last years concerning the stationary state in incompressible homogeneous turbulence of fluid matter, were described already by Dr. Batchelor. I agree with him, that no really decisive argument has been presented so far on the issue in question.

But I think that the physical background against which this whole question has to be seen, has changed considerably since the Paris meeting.

In order to make this clear, I may start from a representation which I believe has been used already by Dr. Batchelor in Paris. Fig. 1 is a schematic representation of the spectrum of turbulence. The range of the abscissae covers most of the range which is of interest to us. With the law of Kolmogoroff, Onsager and von Weizsäcker a spectral distribution of the kinetic energy density E_k is found, which is $\sim k^{-5/3}$ in its intermediate range. According to the picture under discussion at that time, energy was fed into the system at low wavenumbers, say by differential galactic rotation. This energy was gradually being shifted to higher wavenumbers by the action of the nonlinear terms in the equations of hydrodynamics and finally dissipated by viscosity at the right end of the scale.

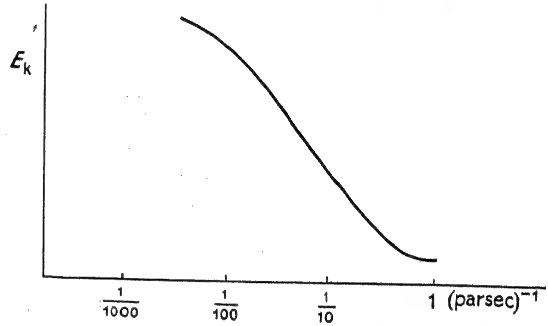

Fig 1. Schematic spectrum of turbulence

The exchange of energy between the magnetic field and the turbulence depends on terms of the form $H_i H_k\,(\partial u_i/\partial x_k)$ in the energy equation. According to Batchelor's picture only in the high wavenumber range there would be equality (as to the order of magnitude) between the energy density of turbulence and that of the magnetic field; in our picture the magnetic energy density would approximate that of the turbulence at much smaller wavenumbers. The interplay of the various spectral components of the velocity field and of the magnetic field has been discussed in a recent contribution to the treatise on cosmic radiation edited by Heisenberg [3].

Now it has become evident since then, that several quite different processes contribute significantly to the inflow of energy and that these operate at different ranges of wavenumbers. Prof. Oort and Dr. Schlüter will discuss tomorrow the expansion of regions heated by high temperature stars (early-type stars); in H II regions of average density the high temperature will produce a large excess pressure, and it is to be expected that every visible H II region must expand. This is a very

efficient source of energy. The early-type stars themselves have high velocities and the distribution of the heated regions is continually changing. For an early B star the diameter of the surrounding H II region can be about 10 parsec; for an O star it will be much larger. The energy density will be of the order 10^{-11} erg/cm^3 or larger, and there will be inflow of energy in *all* regions of the kinetic energy spectrum. It seems likely at present, that the amount in question is much larger than that provided by differential galactic rotation at smaller wavenumbers. Also in the wavenumber range around 1 (parsec)$^{-1}$ and above, the effect of stellar corpuscular radiation has probably to be taken into account. This subject will be discussed on Friday afternoon.

With respect to the mechanism by which the system loses its kinetic energy it must be observed that the dissipation of energy is perhaps most effective in shock fronts: here energy is transferred as it were in a jump from any part of the spectrum to the region of very high k-values. It is possible that this process leads to additional deviations from Kolmogoroff's law, which was enunciated for incompressible turbulence. If the action of the expanding H II regions of one particular size (say 30 parsec) would effectively predominate and if this would result in a spectral distribution of E_k with a pronounced maximum say in the range $\frac{1}{100} - \frac{1}{10}$ (parsec)$^{-1}$, one would have to some approximation the situation considered in Fermi's paper of 1949; it would seem, that in this case the general arguments presented in Alfvén's, Elsasser's and our own papers would lead us to expect a value of H of the order of 10^{-5} Gauss ($H^2/8\pi \approx 10^{-11}$ erg/cm^3).

Evidence for the presence of magnetic fields can be derived from considerations on the properties of primary cosmic radiation and its origin. Since the pressure of the cosmic radiation is approximately 10^{-12} dyne/cm^2, the magnetic fields cannot be much less than 10^{-5} Gauss (cf. e.g. [4]).

As a further evidence we may consider the polarisation of the light of distant stars (probably by paramagnetic particles) as observed by Mikesell and Hiltner and the presence of cirruslike structure in nebulae of small dimensions. In the Pleiades nebulae the diameter of the filaments is of the order of 10^{16} cm; without magnetic fields the presence of such details cannot well be understood.

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CHAPTER 26

DISCUSSION ON INTERSTELLAR MAGNETIC FIELDS

Chairman: Dr. TH. VON KÁRMÁN

LIEPMANN: Why is it that Kolmogoroff's law does not hold in some nebulae?

BIERMANN: The turbulent spectrum depends on the detailed mechanisms by which kinetic energy is being fed into the system, especially on the spectral distribution of these sources of kinetic energy. In view of what has been said before, there may be considerable differences between different nebulae and especially for isolated clouds. The spectrum of an individual nebula may therefore have no simple form at all.

COWLING: In much of the work described today the main interest is not in the results derived so much as in the initial assumptions. For example, in Chandrasekhar and Fermi's work the interest, and the difficulty, is in the assumptions that a magnetic field acts along a spiral arm, and that the spiral arm is a mass which would collapse gravitationally in the absence of the field. Bullard's initial assumption is that a steady field can be produced by a steady motion, and his eigenvalues are the strengths of the velocities required. If one postulates the same or some other steady motion and sees how the field varies, one can expect to get complex eigenvalues corresponding to fields increasing like a complex exponential in the time, i.e., like an increasing oscillation. Such fields resemble more what one gets in turbulent motion and may present difficulties in interpretation. Biermann's assumption is that equipartition of energy exists between turbulent motion and a magnetic field. I confess I cannot see why this should be so. Just as the non-linear terms in the hydrodynamical equations tend to transfer energy from longer to shorter wavelengths in the turbulence spectrum, so the non-linear terms in the equation for $\partial H/\partial t$ should transfer magnetic energy from longer to shorter wavelengths. Thus while some magnetic energy may be fed into the longer wavelengths it may be doubted if equipartition with the mechanical energy can be reached in these.

BIERMANN: In reply to Cowling I should like to remark that there is

no "autochtonous" transfer term (like the v^3/l term) in the equations for the magnetic field. The energy transfer from one wave number to another is only via the velocity field. So every single part of the H-spectrum is linked with the corresponding part of the v-spectrum.

SCHATZMAN: I have been extremely interested by Lighthill's paper. However, although his discussion of compressibility effects can be applied e.g. to stellar atmospheres, it does not apply to the interstellar gas.

LIGHTHILL: I agree that all my rigorous results refer to small root mean square Mach numbers (< 1) and thus do not apply to the interstellar gas. But perhaps the conjectural remarks at the end of my paper may be relevant.

THOMAS: I do not understand Schatzman's remark. It seems to me that Lighthill's discussion is very particularly relevant. It is just this problem of dissipation of energy by compressibility effects which is important here, and to which we have been so long in coming. By far the greatest application of the term "turbulence" to astronomy has been through a rather blind borrowing of the results obtained in the special case of the incompressible, isotropic case. And, as Biermann just remarked, the spectrum must certainly depart from this Kolmogoroff result when any dissipation by shocks occurs. Certainly the occurrence of Mach numbers much greater than those for which Lighthill's results remain strictly valid, do not invalidate the physical tendency of these results—one can only expect greater dissipation of energy, not less. Indeed, in 1947 I suggested that a field of supersonic turbulence in the ordinary sense represented a physical contradiction; for one would expect a field of shock waves to appear and dissipate energy very rapidly. One rather needs now a physical theory of turbulence, an inquiry into detailed processes rather than a theory independent of processes.

FRENKIEL: A preliminary analysis of the spectrum of turbulence in the photosphere of the sun (F. N. Frenkiel and M. Schwarzschild, Astrophys. Journal **116**, 422–427, 1952) has revealed that there are two maxima for the turbulent energy, one at relatively small wave numbers, the other at large wave numbers. This could be explained as a result of two different driving mechanisms introducing two scales of turbulence. It may be possible that a somewhat similar spectrum is obtained in the case to which Dr. Biermann refers. One would then not expect the validity of isotropic laws of turbulence at the intermediate wave numbers, even when incompressible turbulence is assumed, but only for the part of the spectrum at large wave numbers at which the viscous dissipation is of major importance.

BATCHELOR: An additional maximum does not follow from Biermann's remarks.

BIERMANN: This is true. In a region of high dissipation the spectrum cannot have a maximum. But in a general way it may be said that the form of the turbulence spectrum will depend entirely on the spectral distribution of the sources of energy, and that in compressible turbulence the shock waves give an additional deviation from the situation considered by Kolmogoroff.

GOLD: Some consideration must be added to the discussion of gas dynamics in the presence of magnetic fields. The most important departure from the straightforward theory seems to me to arise from the fact that the galactic gas is not necessarily in a single stream flow. There may be several streamlines through each point at each instant, for the galactic gas is quite porous to fast particles. The electro-magnetic effects associated with the slow bulk motion of the material will cause an interaction with fast particles which may occasionally be in the sense of accelerating them, but which may be frequently of importance to the dynamics of the system.

We know that high-speed particles exist in the form of cosmic rays, and one important theory of their origin, namely Fermi's, indeed attributes them to the electro-magnetic effects of the gas masses. One can look at Fermi's theory in a more general way, and one can see that the motion of the gas masses and their associated magnetic fields will, in general, result in electric fields along arbitrary lines. Some of those arbitrary lines will correspond to particle trajectories along which a particle would be accelerated. Those arbitrary lines which are not trajectories are of no interest, and those that correspond to deceleration will contribute to the decay of cosmic ray flux.

The known cosmic ray flux by itself would, through its electromagnetic interaction, affect but not dominate the dynamics of the gas. But in addition there may be more material that is double streaming with the gas, though at slower speeds than cosmic rays and yet fast enough to penetrate some clouds. Such material might provide a means of dissipation of magnetic and kinetic energy far faster than the mere effect of viscosity or ohmic dissipation.

PART V

FORMATION OF COSMIC CLOUDS AND GALAXIES

CHAPTER 27

PRELIMINARY REMARKS ON THE DYNAMICS OF THE INTERSTELLAR MEDIUM*

BY

A. SCHLÜTER**
Göttingen

In discussing the dynamics of the interstellar medium it seems useful to distinguish three types of regions as shown in the following table.

Type of region	log N (atoms/cm^3)	log T (1 °K)	log P (dyne/cm^2)	Relative volume
Dense H II regions	1	4	–10.5	$^1/_2$%
Dense H I regions (clouds)	1	2	–13	5%
H II Regions between clouds	–1	4	–13	95%

The figures have been taken from the report presented at the Paris symposium of 1949; they refer presumably to the region near a spiral arm of our galaxy and not too far from the galactic plane. Incidentally, these figures lead to a rather low value for the total mass of interstellar matter. If the volume concerned, including the regions between spiral arms, may be taken as $1000(\text{kps})^3 = 10^{67.5}\,\text{cm}^3$, with an overall mean density of $10^{-24.2}$ gr/cm^3 (1 atom per 3 cm^3), a total mass of $10^{10}\,M_\odot$ would result.

Since 1949 the low value of the temperature of the H I regions has been confirmed by measurements of the 21-cm line. Hence the following simplified picture suggests itself now: In the H I clouds and the hot regions between the clouds there is an internal pressure of very roughly the same order of magnitude ($\sim 10^{-13}$ dyne/cm^2) and the pressure in the dense H II regions (some 10^{-11} dyne/cm^2) is much larger. On

* This paper was adapted from part of the Preliminary Communication No. 10, prepared in advance of the symposium. An extended version has since been published. *The Editors.*

** Prepared in collaboration with L. Biermann.

account of this pressure difference there does not seem to be any escape from the conclusion* that all observable, i.e. dense, H II regions must expand very rapidly. The velocity may be of the order of 10 km/sec. This expansion is not adiabatic, but nearly isothermic, and ionizes—as is easily verified—a steadily increasing mass of gas, as long as this process goes on in an H I cloud.

If the ionizing stars would remain equally powerful and fixed in space, the expansion would probably lead to a final situation in which all ionized regions would have a density of 0.1 atoms per cm^3 and the pressure would be nearly equal in all space. The mutual motions of the stars, of the order of 10 km/sec, and the fact that the hottest stars have short life times, of the order of 10^7 year, make the actual situation quite different. The main effect to be expected is a continuous redistribution of interstellar matter. The mean distances of the O- and B-stars from the galactic plane are indeed of the same order as the mean distance of the gas from the galactic plane. A second important consequence is that the motions of the interstellar gas are supplied with energy from stellar radiation, i.e. from nuclear reactions.

An estimate of the energy, converted in this way into mechanical energy of the interstellar material, gives 10^{35} erg/sec for an H II region around a B 0 star. On the other hand, 10 typical H I clouds corresponding to the one ionized may have a mass of 10^{36}—10^{37} g. Thus an energy supply of 10^{-2}—10^{-1} erg/g.sec is found, which seems easily sufficient for maintaining the irregular "turbulent" motion of the intersteller material and at the same time its irregular density distribution.

It appears likely that the dissipation of this energy goes on mainly (but not exclusively) at the boundaries of the expanding dense H II regions and at those of the H I clouds. In the language of the spectral theory of turbulence this means that energy may be transferred discontinuously from small to high wavenumbers. This is in contrast to the picture previously suggested on the basis of the theory of the turbulence of incompressible fluids.

A final consequence of the necessary continual expansion of every dense H II region is that it appears most unlikely that an O- or B-star can regenerate its hydrogen from the dense interstellar material around it and thus be rejuvenated. Only the alternative, that early type stars are steadily being born, seems to be consistent with the argument just given.

In addition to the main effect discussed above, two further effects

* This conclusion was first drawn by Spitzer at the Symposium at Paris, August 1949.

have to be discussed, namely radiation pressure and the pressure exerted by corpuscular radiation.

The influence of *radiation pressure* on smoke grains has been discussed on the basis of the Mie theory by many authors. One aspect is the diminution or overcompensation of gravity near a star, another the apparent attraction of any two grains in a more or less uniform radiation field. The pressure of the general interstellar radiation field, acting e.g. on the surface of an obscuring cloud is 10^{-12} dyne/cm^2; it is higher only near overluminous stars.

In interstellar space radiation pressure acts on atoms or molecules practically only via resonance transitions. Most important is the pressure in the Lyman continuous radiation and in the Lyman α line*. At the boundary of a normal H II region around a B 0 star its amount is again 10^{-12} dyn/cm^2 for a distance of ≈ 6 psc, corresponding, according to Strömgren's relation, to 10 atoms/cm^3. This means only a very small correction to the gas pressure causing the expansion. On the other hand, the radiation pressure acting on the surface of an H I cloud surrounded by a low density H II region will be about 10^{-13} dyn/cm^2. This is at least as much as the gas pressure and provides an additional stabilizing agent for the H I clouds.

The *corpuscular radiation* of main sequence stars may have the following effect. If the fraction α of the luminosity L of a star is emitted in addition to thermal radiation in the nonthermal form of corpuscular radiation, say of velocity $v = \beta c$, then the momentum transferred by this latter per second is of the order of α/β of that transferred by light, if v is of the order of the escape velocity, but somewhat larger. Both for α and β the range 0,001 ... 0,01 appears plausible. While the total pressure exerted in this way is comparable with that of light (if it becomes absorbed), the density of the resulting volume force may be much higher owing to the large cross sections involved.

The *pressure exerted by expanding shells of novae* and similar objects presents, from the hydrodynamical point of view, a similar problem. Further discussions of this subject will probably have to take into regard the possible effects of interstellar magnetic fields.

The whole action of an early B- or O-star on an interstellar cloud presents an interesting problem of shock wave theory. Also the influence of the gravitational field and the radiation field of the galaxy on the geometrical structure of an expanding H II region would seem to deserve attention.

* Discussed in detail by L. Biermann and A. Schlüter, Z. f. Naturf. 9a, 463 (1954).

CHAPTER 28

OUTLINE OF A THEORY ON THE ORIGIN AND ACCELERATION OF INTERSTELLAR CLOUDS AND O-ASSOCIATIONS*

BY

J. H. OORT
Leiden

1. LARGE CLOUD COMPLEXES AND THEIR DISRUPTION

Evidence presented by the interstellar absorption lines and by observations of dark clouds has led to the conclusion that interstellar matter occurs largely in discrete clouds. For a working model an average cloud might be estimated to have a diameter of about 10 parsec and a density of the order of 10 hydrogen atoms per cm^3, corresponding with a mass of the order of 100 solar masses. Here and there the clouds form great accumulations, where average densities of 10 or more are found over regions several tens of parsec in diameter, while locally the density sometimes runs up to values of considerably more than 1000 H/cm^3. The existence of such large cloud complexes has been pointed out by several authors. An example is the agglomeration of dark clouds in Taurus, where we may estimate that the interstellar density is about 10 times the average over a region of some 60 parsec diameter. The total mass of

* Since the Cambridge Symposium the suggestions concerning the origin of the interstellar clouds and their acceleration presented at that time have been considerably elaborated. The article presented here differs, therefore, in several respects from the communication made at the Symposium. It has appeared in a somewhat extended form in the Bulletins of the Astronomical Institutes of the Netherlands (No. 455), while another article on the same general subject, by Oort and Spitzer, is to appear in the Ap. J.

The conception of rocket-like acceleration of interstellar clouds originated in discussions with Spitzer during a stay in Princeton in 1952. The idea that O-stars would be responsible for the replenishment of kinetic energy in the interstellar medium has been proposed independently by Biermann and Schlüter at the Cambridge Symposium (see the preceding communication, Ch. 27).

this cloud complex would be some 30 000 solar masses. Most of the matter is non-ionized and presumably at a temperature of the order of 100 °K.

An example of somewhat different type is shown by the nebulae around the open cluster NGC 2244 in Monoceros, which have recently been described by Minkowski [1]. They form a luminous mass of roughly circular shape, with a radius of 40′, corresponding, at the estimated distance of 1400 parsec, with 16 parsec. The surrounding dark clouds, which are evidently connected with it, show that the bright nebula is only a part of a considerably larger mass. The mass of the luminous nebula is estimated by Minkowski as 10 000 solar masses. That of the whole cloud complex is likely to be several times larger; of the same order, therefore, as the mass of the Taurus clouds. The luminosity of the nebula is due to the ultraviolet radiation of four O-type stars in the central cluster, which suffices to ionize the hydrogen up to the edge of the luminous part. It seems reasonable to assume that the O-stars (and the cluster of which they are part) were born fairly recently inside this large mass of clouds, which they must now be disrupting by their powerful ionizing action. The disruption must be relatively rapid, as we can see by considering a somewhat schematic example based on the Monoceros nebula. Following Minkowski, the average density may be estimated from the surface brightness as well as from the radius of the Strömgren sphere*; both give 23 H/cm^3. With a temperature of 10 000° the pressure in the ionized sphere would be 0.63×10^{-10}.

The pressure in the surrounding cool masses must in general be two orders of 10 lower. The cool gases will therefore be compressed. The heat generated by this compression will probably be radiated sufficiently rapidly to prevent a large rise in temperature [2]. It now seems probable that dense cool clouds contain a considerable amount of hydrogen molecules, by which the radiation is enhanced. It is similarly enhanced by the large numbers of solid particles in such clouds. The density in the outer shell will then become of the order of a hundred times that in the hot inner region. Under suitable conditions the compression layer will start moving outward into the cool mass with a velocity of the order of the velocity of sound in the hot inner mass, say 11 km/sec.

While it moves outward the density in the inner mass decreases, and as a consequence additional parts of the un-ionized shell outside will be

* The Strömgren sphere is the sphere of hydrogen that would be ionized by the radiation of the star, or group of stars, if the hydrogen were distributed homogeneously.

ionized. As the newly ionized particles leave the cool shell in the direction of the inner mass their escape will exert an outward force on the remaining part of the cool shell. To compute the ensuing motions and density distribution in the cold and hot masses is an intricate problem *. For the case of a radiation front moving into a flat cloud this probleem has recently been partially discussed by Dr Kahn [3]. For the present rough survey I shall limit myself to an estimate of the total momentum that will be gained by the cool shell as a consequence of the ionization and subsequent escape of hydrogen at the surface facing the O-star.

It can readily be seen that the gravitational attraction exerted by the total mass is of slight importance. Let us suppose the luminous nebula to be surrounded by a cool mass of the same density and with a 1.5 times larger radius. If we consider a cone with its top in the centre of the nebula, and having a cross section of 1 cm^2 at the surface of the bright nebula, this cone would contain a total mass $m_n = 1.5 \times 10^{-3}$ g of non-ionized material. The gravitational attraction on this cone would correspond to a total force of 1.0×10^{-12} dynes/cm^2 at the inner surface of the cool mass. This is 60 times smaller than the gas pressure in the hot inner mass, and can therefore be neglected.

In order to obtain a rough estimate of the fraction of the neutral shell that will be ionized during the process of the expansion we suppose the ionized gas to be evenly distributed and to have at each moment such a density that the inner radius of the cool shell will be equal to the radius of the Strömgren sphere. It can be shown that in most actual nebulae there is sufficient time to enforce the latter condition. In reality the inner mass will not be homogeneous, but it is probable that the supposition made will indicate at least roughly what will happen to the cool shell.

Suppose an O-star to have formed in the central part of a spherical nebula of density n_0 hydrogen atoms per cm^3 and radius R. If we denote by s_0 the radius that the Strömgren sphere around this O-star would have if it were situated in a homogeneous medium with a density of 1 hydrogen atom per cm^3, the actual radius of the Strömgren sphere at the time of the birth of the O-star would be $r_0 = s_0 n_0^{-2/3}$. Consider now a later stage, at which the cool shell has been completely compressed and has grown to a radius r exceeding the original outer radius R. Calculations given elsewhere show that the time available is sufficient to establish a quasi-equilibrium, such that the new density n corresponds again to a radius of the Strömgren sphere equal to r; hence

* Schatzman and Kahn have considered the development of the shock wave in a case like the one treated here, without taking account of radiation (cf. p. 163).

$$r = s_0 n^{-2/3}. \tag{1}$$

The total mass of the ionized sphere is now $\frac{4}{3}\pi m_H n r^3$, or $\frac{4}{3}\pi m_H s_0^{3/2} r^{3/2}$, if n is eliminated with the aid of (1). Originally it was $\frac{4}{3}\pi m_H n_0 r_0^3$. The difference must be equal to the diminution of the cool shell's mass. If, as illustrated in Fig. 1, we denote the original mass of the cool shell by

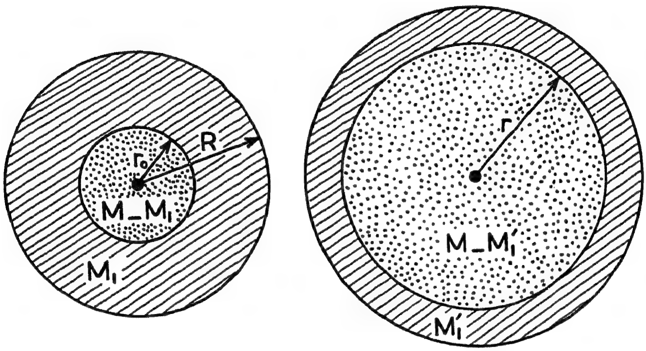

Fig. 1. Schematic drawing of nebula before expansion and after a certain time of expansion. The total mass, M, remains the same.

M_1, the new value by M_1' and the *total* mass of the nebula by M (including the ionized part), we have evidently

$$\frac{M_1'}{M_1} = \frac{M - \frac{4}{3}\pi m_H r^{3/2} s_0^{3/2}}{M - \frac{4}{3}\pi m_H n_0 r_0^3} = \frac{1 - \frac{4}{3}\pi m_H r^{3/2} s_0^{3/2} M^{-1}}{1 - (r_0/R)^3}.$$

For the present we shall only consider large nebulae, for which, at the time of the birth of the O-star, the radius exceeded considerably the radius of the Strömgren sphere in the dense nebula. In the above expression for M_1'/M_1 the denominator may then be put equal to 1, and we have, approximately,

$$\frac{M_1'}{M_1} = 1 - \frac{\frac{4}{3}\pi m_H r^{3/2} s_0^{3/2}}{M}. \tag{2}$$

The hydrogen that is being ionized at the inside of the cool shell will fly off in the general direction of the O-star. It disperses into a region in which the density is considerably smaller than in the cool shell. The momentum it carries away must be compensated by an increase of the outward velocity of the remaining shell. The shell will be accelerated like a rocket. This mechanism has been studied in a paper by Spitzer and

Oort, which is to appear in the Astrophysical Journal. If the average velocity along the radius vector, with which the particles finally escape, is called V, and if v and v_0 denote the velocity of the shell, respectively at the time considered and at the time before an appreciable mass had been lost, we have the following equation expressing the conservation of momentum

$$M_1' \frac{dv}{dt} = -V \frac{dM_1'}{dt}. \tag{3}$$

This leads to the solution

$$\frac{M_1'}{M_1} = e^{-(v-v_0)/V}. \tag{4}$$

These formulae as well as estimates of V may be found in the article by Oort and Spitzer just mentioned. I shall adopt $V = 20$ km/sec.

From the expression for the mass of the ionized sphere given above we see that when the radius increases by dr the amount of mass lost by ionization is $2\pi m_H s_0^{3/2} r^{1/2} dr$. As the average amount of "ordinary" interstellar matter (of density $n_H = 1$) swept up during the same time interval is $4\pi r^2 m_H dr$, we see that the two become equal when r has grown to $2^{-2/3} s_0 = 0.63\, s_0$. If we wish to stop our calculation of the acceleration before interaction with interstellar matter becomes serious, we should stop at about half this radius, say at $1/3\, s_0$. The growth by interstellar matter is then 38 % of the loss by ionization. In the rough estimates that follow we shall suppose that the acceleration continues up to this point unhindered by interstellar matter, and we shall use the velocities obtained at this distance to estimate the amount of momentum created by the birth of the central O-star. This procedure is likely to give a slight but not serious under-estimate, because some additional acceleration will occur beyond this distance.

If we denote by M_t the mass that can be ionized when the density has decreased to the point where the radius of the Strömgren sphere has become $1/3\, s_0$ it is clear that if the original mass of the nebula is smaller than M_t it will evaporate entirely before it has reached this radius. Under the assumptions made above, M_t is therefore the critical mass. It is equal to $M_t = \frac{4}{3}\pi m_H n r^3$ where n is given by (1) and $r = 1/3\, s_0$, so that

$$M_t = \frac{4}{3}\pi \times 3^{-3/2} m_H s_0^3. \tag{5}$$

This is equal to the numerator of the second term in the right-hand

member of (2) for $r = 1/3\, s_0$. For the nebula surrounding NGC 2244 Minkowski found that the ultraviolet radiation from the O-stars contained in it would correspond to a value of s_0 equal to 130 parsec. In this case $M_l = 43\,000$ solar masses.

If the mass of the nebula is larger than M_l, part of it will fly off in the form of neutral clouds. It is clear that while the "radius" of the shell grows it must, by its inherent irregularity as well as by the sweeping up of irregularly distributed interstellar matter, break up into separate parts that will evidently have greatly varying sizes and forms, but will all be very flat. We may identify these fragments with newly formed "ordinary" interstellar clouds. It is true that these parts will not in general be stable structures; they will keep on growing in lateral direction much the same way as the "shell" was growing. But the separate interstellar clouds that we observe are not stable either. This is evident from their shapes; moreover there is some indication that considerable internal motions may exist [4].

If the mass thickness exceeds the critical value corresponding to M_l, the neutral mass remaining at $r = \frac{1}{3} s_0$ is equal to $M—M_l$. Its velocity may be calculated from (4). The results are given in the following table. The last column shows, for central O-stars like those in the Monoceros nebula, the total momentum of the remaining neutral shell expressed in units of solar mass and km/sec. I have taken $v_0 = 11$ km/sec, $V = 20$ km/sec.

Total mass ($\approx$ original neutral mass) $M \approx M_1$	Final neutral mass M_1'	Ratio M_1'/M_1	Final velocity (km/sec)	Momentum of final neutral shell (solar masses × km/sec)
1.1 M_l	0.1 M_l	0.091	59	2.5×10^5
1.2 ,,	0.2 ,,	0.167	47	4.0 ,,
1.4 ,,	0.4 ,,	0.286	36	6.2 ,,
1.6 ,,	0.6 ,,	0.375	31	8.0 ,,
1.8 ,,	0.8 ,,	0.444	27	9.3 ,,
2.0 ,,	1.0 ,,	0.500	25	10.8 ,,

It is not possible to estimate with any confidence the total mass of the Monoceros nebula. For the luminous part Minkowski gave a value of 10 000 solar masses. The total mass of the cloud complex including the cool mass must be several times larger; probably, therefore, of the order of the value of M_l given above. As the nebula is highly irregular, there

would thus be parts that have a rather greater mass thickness than that required to survive as neutral clouds. Other parts would, however, be entirely ionized. Although the above analysis cannot, strictly, be applied to this more irregular case, we may obtain an order-of-magnitude estimate of the amount of interstellar motion generated in the Monoceros nebula by assuming that the above numbers may also be used when dealing with *parts* of shells. If we assume, tentatively, that 50% of the shell has a mass thickness exceeding the critical value, and that in these parts the mass thickness would lie mostly between that corresponding to M_l and twice this value, and be evenly distributed over this range, we find that the escaping neutral clouds would have a total momentum of 3×10^5 solar masses $\times$ km/sec, or a kinetic energy of 5×10^6 solar masses $\times$ (km/sec)^2. The weighted average velocity would be 31 km/sec.

In order to obtain an estimate of the number of "clouds" that would be formed from a shell like that surrounding NGC 2244, let us define an interstellar cloud somewhat arbitrarily as a compact stretch of gas in which the velocities at the extremes do not differ by more than 3 km/sec. Such a relative velocity would correspond to a maximum growth of the lateral dimensions by 10 parsec during the mean life time of a new cloud (i.e. the average time before it will collide with another cloud, which may be estimated at three million years). If we now take 30 km/sec as the final expansion velocity to be obtained, a velocity variation of 3 km/sec would occur over an angle of 6° as seen from the exciting stars. The matter contained in a solid angle of 28 square degrees may then be considered as forming an individual cloud. If we suppose, somewhat arbitrarily, that the half of the original mass that had sufficient thickness to survive, was contained in a quarter of the sphere, or 10 000 square degrees, the shell would form some 350 clouds in the sense just defined. Each of these would have a mass of roughly 30 solar masses. This is somewhat smaller than the estimate quoted in the first paragraph of this article, but both estimates are uncertain by a larger factor than this difference. For the present, order-of-size agreements are all that we can reasonably ask for.

The case of the Monoceros nebula is by no means unique. Similar conditions are found in all the well-known emission nebulae, like Messier 8, Messier 16, Messier 20. In all these cases the cloud complexes appear to be so large that they could only partly be ionized by the associated O-stars. The surrounding cool masses must be compressed as wel as accelerated outward from the O-stars. The data at our disposal are entirely insufficient to determine in any of the objects mentioned what

the final velocities would become. As a rough guess we might estimate that those neutral clouds which escape complete ionization will do so with an average velocity somewhere between 20 and 40 km/sec. In some cases, however, rather higher velocities may arise.

The interesting thing is that the mechanism considered puts concentrated interstellar matter into co-ordinated motion over stretches comparable in dimension to interstellar clouds. It is of interest to inquire whether the amount of cloud motion furnished by this process could be sufficient to explain the observed frequency of interstellar clouds and their motions. The principal condition to be fulfilled is that the frequency of the process be high enough to counterbalance the loss of kinetic energy of clouds by mutual collisions.

2. BALANCE BETWEEN FORMATION OF MOVING CLOUDS AND LOSS OF ENERGY BY COLLISION

We may get some idea of the extent of the process by considering the large and bright emission nebulae which are presumably the main contributors. The following five major nebulosities north of −25° declination may be classed in this category: The combined nebulosities in Orion, the Monoceros nebula surrounding NGC 2244 (sometimes called Rosette nebula), Messier 8 (including the neighbouring, but much smaller Trifid nebula, or Messier 20), the North-America nebula and the nebulae near γ Cygni. It is possible that the last two are connected with each other. Our knowledge of the southern nebulae is still somewhat incomplete. In each of the above cases the mass of the luminous nebulae must be of the order of 10 000 times the sun's mass, except in M 8, where it is only a few thousand solar masses. In all cases there are considerable dark masses connected with the luminous nebulae, the cloud complexes being evidently larger than the amounts of hydrogen that can be ionized by the embedded O-stars. The distances of the nebulae considered are all less than 1500 parsec. In order to get an order-of-size estimate of the amount of kinetic energy imparted to the interstellar medium I shall assume that within 1500 parsec there are 5 cloud complexes of the same order as the Monoceros nebula, each yielding a kinetic energy in the form of translational motions of cool clouds amounting to 5×10^6 in units of solar mass $\times$ (km/sec)^2. It may be estimated that the whole process of acceleration and dissipation of a great emission nebula will not take more than two million years. With an average life of two million years for these nebulae the total gain of energy in the form of cloud motions would thus be 12×10^6 units per million years.

In order to find the loss by collisions we take Blaauw's estimate according to which a line of sight of 1 kps length would cut 10 clouds on the average. If the clouds have a mean diameter of 10 parsec there must then be 130000 clouds per kps^3, or about 200000 within a cylinder with axis perpendicular to the galactic plane and with a radius of 1.5 kps, if the effective thickness of the gas layer is taken 250 parsec. With a mean velocity of 10 km/sec, or 10 parsec/million years, a cloud will have a probability of 1/10 per million years to collide with another cloud in such a way that the centres of the two clouds pass at a distance less than their radius. We may assume roughly that in these collisions about half of each cloud will take part in the collision, the resultant mass obtaining an average velocity $1/\sqrt{2}$ times that of the original clouds. The other halves will retain their original energy. Per cloud of mass 100 suns the loss of kinetic energy would be 1200 solar masses $\times$ (km/sec)^2. Per million years, in the cylinder considered, 20000 clouds will suffer a collision of this kind. The actual loss per collision may be less because collisions that are not in the direction of the interstellar magnetic field may be partly elastic. Neglecting this factor for the present, the resulting total loss of energy of cloud motion would be 24×10^6 solar masses $\times$ (km/sec)^2.

The fact that this number is of the same order as the estimated gain suggests that here is at least a possibility for explaining the cloudy structure and the large internal motions in the interstellar gas. In view of the enormous uncertainty in the estimates it should not be considered to be more than a *suggestion*. The greatest uncertainty is in the masses of the non-ionized parts of the large cloud complexes. These might be considerably smaller than was assumed, in which case little non-ionized matter would remain after their expansion. However, also the expanding *ionized* masses will be able to transfer translational energy to cool clouds. A rough estimate shows that in the case of the Monoceros nebulae the translational kinetic energy of the ionized parts would be of the same order as the value that was assumed above for the cool parts. Something between $\frac{1}{4}$ and $\frac{1}{2}$ of this energy will be imparted to cool clouds in the surrounding space. If it is supposed that the nebulae are ultimately completely ionized, the actual gain may thus be smaller than that found above. On the other hand there are in the space within 1500 parsecs many more O-stars than those contained in the 5 large cloud complexes considered. These other O-stars will probably also have had a certain share in accelerating interstellar gas. That this share may be appreciable is indicated for instance by the occurrence of a high-velocity cloud of

considerable mass connected with the very near-by and young association of stars around ζ Persei (see 3. below).

We may now imagine the following tentative picture of the cyclical process of formation of interstellar clouds. The existing, ordinary clouds continually lose energy by sweeping up intercloud material of low density, and through mutual collisions. By the latter they are partly cut up into smaller clouds. On the other hand we may suppose that the slower-moving clouds sometimes combine, perhaps aided by gravitation, into larger ones. Once an important mass has been formed it will tend to absorb further slow clouds moving into it, and develop into a continuously growing agglomeration. The observed emission nebulae give evidence that at some time or other, somewhere in such a cloud complex conditions become favourable for the formation of one or more O-type stars. This marks the end of the growing cloud complex, which is exploded by the large-scale ionization of the hydrogen due to the ultraviolet radiation of the O-stars. The cool masses pushed away by this explosion then put mass and motion into the neighbouring clouds. The suggested theory is evidently very incomplete in many points, in particular in so far as the statistics of cloud collisions and the growth of large aggregates is concerned.

The picture drawn above receives rather direct support from two classes of data, namely the observations of interstellar absorption lines and the expansion of stellar associations.

The interstellar absorption lines observed in stars of two large associations of early-type stars, in Orion and Lacerta, show components with considerable negative velocities which can most plausibly be interpreted as arising in shells of interstellar gas surrounding these associations and expanding with velocities up to about 20 km/sec. The preponderance of high negative residual velocities over positive ones, which has been noted by several authors to be a general feature of interstellar absorption lines, may well be due to the same cause.

3. THE FORMATION OF EXPANDING O-ASSOCIATIONS

A remarkable confirmation that large-scale expanding motions are started in the vicinity of O-type stars is found by the study of the motions of young stars.

A few years ago Ambartsumian has developed the theory that W-, O- and B-type stars form, at least in large part, expanding groups, or associations, as he called them [5]. He advanced evidence for the fact that

these associations are inherently expanding, independent from any shearing effects caused by differential rotation of the Galactic System. The existence of such expansions found a direct confirmation through the work of Blaauw in Leiden, who was, moreover, able to determine the amount of the expansion in several cases [6]. It can become quite high, of the order of 15 km sec for the association surrounding ζ Persei.

Calculating backwards Blaauw infers that the stars in the latter group must have started 1.5 million years ago from a much smaller volume of space than they now occupy. The most direct interpretation would be to assume that the stars have actually been formed from the interstellar gas at that epoch. But why did they start moving away from each other, and with such high velocities?

It seems indicated to seek the answer in the expansional motions that, as we have just considered, must be set up in the cold regions surrounding the ionized nebula around a newly born O-star (or group of O-stars). We do not know how these primary O-stars were formed; we have to accept this as a fact of observation. But it is tempting to think that subsequently the strong compression in the surrounding cool clouds will lead to the formation of new early-type stars that then would share the outward motions that the ionized region has imposed upon these clouds. The expansion of the associations of early-type stars would in this way become understandable.

This picture of the origin of the expanding motion of the associations finds some visible support in the chain-like dark regions of small angular dimension and presumably very high density that are found in several of the large emission nebulae. As an example I may once more refer to Minkowski's article on the nebulae around NGC 2244.

In the NW quadrant of this nebula we find a chain of small, irregular dark clouds in which, judging from their dimension and absorption, the density is of the order of 10^4 atoms per cm^3. They give the impression of having been formed by compression of a cool nebula by the ionized region. That stars may be forming in this compressed region is indicated by the presence in just this part of the Monoceros nebula of numerous so-called globules: detached, roundish, dark patches of minute dimensions. There is some ground for the supposition, made by Bok and others, that such globules represent an early stage in the process of birth of stars. The density estimated for the chain of dark clouds mentioned seems to be of the right order to make possible contraction of the interstellar matter into stellar objects in times comparable to the presumable ages of expanding stellar associations.

If our hypothesis concerning the origin of the expanding associations of stars is correct, there should be two categories of early-type stars, *viz.* the primary O-type stars formed once in a long time when, accidentally, somewhere in a cloud complex sufficiently favourable circumstances for spontaneous star formation occur, and, secondly, the secondary early-type stars formed in the large compressions caused in the cool cloud complex as a consequence of the expansion of the ionized region surrounding the primary O-stars.

If the primary O-stars are formed in a region of very irregularly distributed matter, it may happen that isolated dense clouds obtain quite high velocities by the rocket effect due to the continued ionization and "evaporation" of hydrogen from the face directed toward the O-stars. It is possible that in this way the remarkably high velocities shown by a number of O-type stars as well as by some interstellar clouds can be explained [7].

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CHAPTER 29

DISCUSSION ON THE EXPANSION OF COSMIC CLOUDS AND ON LUMINOUS FRONTS

Chairman: Dr. T. G. COWLING

SAVEDOFF: It has been suggested by Oort that the energy of the interstellar medium is derived from the expansion of H II regions around O-stars. This mechanism has already been discussed by Schlüter, but I would like to make some additional comments.

If we consider the idealized problem of the motion engendered by the creation of an O-star in a homogeneous medium, we are first concerned with the time scales of the various phenomena. For the Orion nebula with a density $n_p = 3700\ \text{cm}^{-3}$ and a radius of $\frac{1}{2}$ parsec, we have a region consistent with a Strömgren sphere ionized by the O6 star in the Trapezium. The surrounding larger area is weakly ionized by the general field of early type stars. In the dense parts of the Orion nebula, the ionization time scale, $t \approx 1/(n\,\sigma\,v)$, is about 50 years. This is the time in which the number of protons will decrease a factor e by recombination in the absence of ionizing radiation. Further, since the temperature of an H II region is about 10.000° K, a sound signal can traverse the $\frac{1}{2}$ parsec radius in about 50.000 years. As the Trapezium stars are probably older than this age, we expect to observe the hydrodynamical effects produced by the large pressure difference between the H I and H II regions.

The observations by Campbell and Moore indicate no spherical expansion as we might anticipate. There are indications of a velocity gradient across the face of the nebula. The Orion nebula is apparently situated on the face of an extremely dense H I region for which an opacity of approximately 10 magnitudes per parsec has been estimated. Perhaps this is connected with the peculiar symmetry of the expansion? If the absence of an expansion is to be attributed to gravitational or other forces preventing the expansion of the H II region, one would expect to find a collapse of the surrounding H I region. This is not observed.

I believe the Orion nebula is the only H II region in which detailed velocity measurements have been made. Other regions appear to be too faint for comparable study. Thus the absence of a definite expansion in the Orion nebula leaves us a little uncertain about the process suggested by Oort. I may add that the nebula is not held together gravitationally. The velocity of escape with the estimated mass is 1 km/sec, the thermal velocities of the order of 10 km/sec.

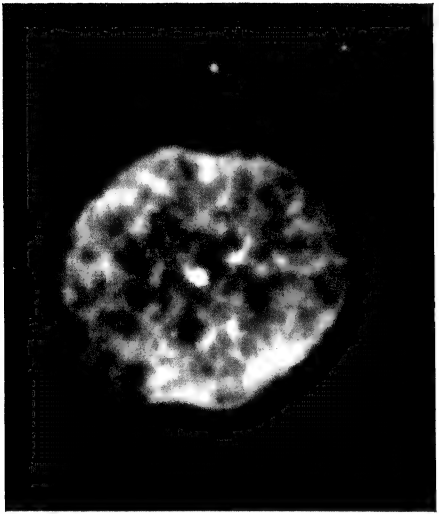
THOMAS: I do not understand Savedoff's comment that no hydrodynamic effects can be present. He has assumed that he suddenly "turns on" the O-star, and in 50 years an H II region $\frac{1}{2}$ parsec in radius results, with a pressure discontinuity of magnitude 200–300 on the boundary. Clearly, O-stars are not created in 50 years—I am not sure how long it takes, but certainly longer than this. Consequently the H II regions builds up slowly, and we should expect any pressure discontinuities to propagate outward through the gas. Indeed, even for 100° K for the low-pressure side of such a wave, a pressure discontinuity of the order mentioned will travel with a speed of 25 km/sec and traverse the $\frac{1}{2}$ parsec in 2.10^4 years—which is still young for an O-star.

I would prefer to see a treatment which considered the gradual evolution of the H II region as the O-star evolved along the path which has been suggested several times at this symposium, a condensation and contraction and gradual building-up of the high emission by the O-star. It would be very surprising to find abrupt pressure discontinuities arising unless it is wished to introduce nova-like outbursts. I believe Oort's characterization of the O-star production as an "explosion" is most misleading—a "low-order conflagration" would be a better term.

SAVEDOFF: I quite agree with you that it is desirable and, indeed, necessary to develop a theory in which the O-star is born gradually, in the time of 10^5 years, say. Sharp discontinuities will persist for a time longer than the time of star formation, when the density is such that the distance sound travels in the time of star formation is comparable to the radius of the H II region. At a later period I would expect approximate equality of pressures which is the condition that characterizes a conflagration.

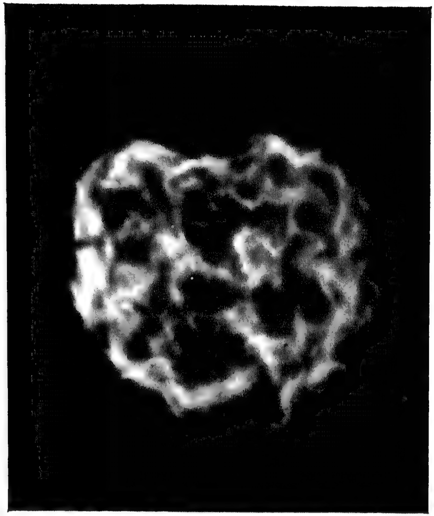
FRENKIEL: When Minkowski showed his photographs of nebulae, I was rather struck by a certain analogy with photographs of exploding sparks obtained in our laboratory*. These experiments consist of observing the behavior of a mass of air heated by the discharge of an

* Due to the courtesy of Dr. Frenkiel and Dr. Minkowski, an illustration has been prepared for these Proceedings and is shown opposite this page. *The Editors*



A

NGC 1501, a planetary nebula of apparently turbulent character. Estimated distance 3400 parsec; estimated diameter 0.9 parsec.



B

Flame propagation in a turbulent stream. Tenfold exposure of flame at 400 microseconds after ignition in a stream of turbulent gas with velocity 50 meters/second. Diameter 7 millimeters.

The similarity of these pictures may not mean a physical similarity of the processes involved.

electric spark, or the propagation of flames from an electric spark placed in a combustible mixture of gases. In both cases we have studied the gaseous expansion which is often accompanied by a shock wave. The discussion of Oort and Savedoff makes me believe that in addition to the similarity between the photographs of electric spark discharges and of nebulae, there may also be an analogy between some of the physical phenomena. If so, it may be possible to plan some laboratory experiments which will be of interest to astronomers. I shall indeed give more thought to such experiments in the future.

BLAAUW: The interstellar Ca^+ ions in front of the Orion stars show strong negative velocities. I hold this a clear indication that most (or at least half) of the Orion nebula is in a state of expansion. What evidence does Minkowski have for the absence of expansion?

MINKOWSKI: In as much as the measurements of Campbell and Moore and Fabry and Buisson showed systematic velocities, it looked more like a rotation than an expansion. To Savedoff's problem I should like to remark that, no matter how fast the O-star starts, the nebula should expand if the O-star is in it now. But there are strong differences in form. Some of these huge nebulae show distinct ring structures (e.g. Monoceros) and others are extremely confused (Orion, Trifid, etc.). There may be a reason for some nebulae not to expand. The Ca^+ clouds may not have the same velocity as the nebula itself.

OORT: I see no support for Minkowski's suggestion. There is fair evidence that expansion occurs also for nebulae that have not a ring structure, e.g. M 16. As to the observations of the Orion nebula, it is possible that a dense cloud stops the expansion in one direction.

MINKOWSKI: I agree that a lateral expansion of a plane sheet, when viewed obliquely, would be an alternative explanation instead of rotation.

ZANSTRA: This is analogous to the dilemma with the ring nebula in Lyra, which for a long time was thought to be rotating, because the distribution of brightness in the spectral line is asymmetric.

KANTROWITZ: I should like to mention the connection between the physical side and the aerodynamical side of Savedoff's problem (opacity and shock waves). If a shock wave moves faster than 2.10^6 cm/sec, then it will produce ionization and make the gas opaque to free-free radiation.

OORT: But the process determining the extent of the ionized region is the ionization from the ground level, the opacity for which is not affected.

KANTROWITZ: If one assumes that the luminous edges are shock

waves, and if one assumes also that the motion of the gas clouds is controlled by regional magnetic fields, then there should be some relation between the direction of the magnetic field and the direction of the luminous lines. Has any such correlation been observed?

McCrea: Have all speakers satisfied themselves that gravitational effects are unimportant? Is it not a fact that the velocities of the stars concerned are predominantly recessional?

Oort: The gravitational effects are negligible except, perhaps, for the very big cloud complexes. The radial velocities of O-stars are positive and negative, entirely at random.

CHAPTER 30

ON THE MOTION OF H I AND H II REGIONS

BY

E. SCHATZMAN and F. D. KAHN
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(1) We consider quantitatively the problem of the motion of H I and H II regions, following the remarks of Savedoff, Schlüter and van de Hulst at this conference. *

(2) *The size of the H II region.* Around an O-star there exists a zone of ionized hydrogen, here called region A, in which the density is n_A atoms per cm^3 and whose size is determined by Strömgren's formula [1]. Its radius r_A therefore varies as $n_A^{-2/3}$. Suppose that the matter in the zone is moving outwards. As the density of matter falls in the zone its radius grows.

Let u_A be the speed of motion of the material at the boundary of region A, which we call the ionization front. In a time interval dt the density changes to $n_A + dn_A$ and the radius of the region increases by $U dt$, where $U = dr_A/dt$.
Now

$$\frac{dr_A}{r_A} = -\frac{2}{3}\frac{dn_A}{n_A}$$

or

$$U = -\frac{2}{3}\frac{r_A}{n_A}\frac{dn_A}{dt}. \tag{1}$$

In this interval the sphere which contains $\frac{4}{3}\pi r^3{}_A n_A$ atoms increases its radius by $u_A dt$, and so

$$u_A = -\frac{1}{3}\frac{r_A}{n_A}\frac{dn_A}{dt}, \tag{2}$$

so that

$$U = 2\,u_A. \tag{3}$$

* We follow closely the derivation presented at the symposium. For later work on related problems see references [2,3].

(3) A shock wave is formed in the H I region surrounding the expanding H II region. Going outwards from the star we have, therefore, regions A, B, C: the H II region which we have discussed, a compressed, and an undisturbed H I region.

Let u_B be the particle velocity, n_B the particle density and p_B the pressure in region B. We have the equation of conservation of matter

$$(U - u_B)n_B = (U - u_A)n_A$$

or, by (3),

$$n_B(2u_A - u_B) = n_A u_A. \tag{4}$$

Using (3) and (4) the condition of conservation of momentum may be written

$$p_A - p_B = n_A m_H u_A (u_A - u_B). \tag{5}$$

Here m_H = mass of the H atom.

(4) *The shock wave.* We have the usual conditions at the shock front, i.e. at the B-C boundary:

Conservation of matter $\quad n_B(V - u_B) = n_C V \quad (6)$

of momentum $\quad p_B - p_C = n_C m_H V u_B \quad (7)$

of energy

$$\frac{5}{2}\frac{kT_B}{m_H} + \frac{1}{2}(V - u_B)^2 = \frac{5}{2}\frac{kT_C}{m_H} + \frac{1}{2}V^2. \tag{8}$$

It is assumed that only the kinetic energy of the particles changes at the shock front.

With p_B as parameter we find the usual formulae:

$$V^2 = \frac{p_C + 4p_B}{3n_C m_H} \tag{9}$$

$$\frac{n_B}{n_C} = \frac{4p_B + p_C}{4p_C + p_B} \tag{10}$$

$$u_B = \frac{3(p_B - p_C)}{[3n_C m_H(4p_B + p_C)]^{1/2}}. \tag{11}$$

Meanwhile by (4) and (5)

$$u_A = \frac{n_B u_B}{2n_B - n_A}. \tag{12}$$

$$p_A - p_B = \frac{n_A n_B m_H u_B^2 (n_A - n_B)}{(2n_B - n_A)^2}. \tag{13}$$

Let $$\frac{p_A}{p_B} = y, \quad \frac{p_C}{p_B} = x\ (< 1), \quad \frac{n_C}{n_A} = \xi\ (\geqq 1).$$

Using (10) and (11) in (13) we find

$$y = 1 - \frac{3(1-x)^2 [4\xi - 1 + (\xi - 4)x]}{[8\xi - 1 + 2(\xi - 2)x]^2} \tag{14}$$

$$u_A = \left(\frac{3\, p_C}{n_C m_H}\right)^{1/2} \frac{(1-x)(4+x)^{1/2}\xi}{\sqrt{x}[8\xi - 1 + 2(\xi - 2)x]}. \tag{15}$$

The model is correct if $2\,u_A < V$, i.e. if the speed of the ionization front is less than that of the shock front. With the aid of (9) and (15), and after a little reduction, the condition becomes

$$\frac{(4x+1)(1-2\xi)}{8\xi - 1 + 2(\xi - 2)x} < 0.$$

When $0 < x < 1$ and $\xi > 1$ this is certainly satisfied.

(5) *Two numerical examples.* Two particular cases are worked out numerically in the table below. The temperatures T_C ($= p_C/kn_C$) and T_A($= p_A/2kn_A$) are taken to be 100 °K and 10 000 °K, respectively. The factor 2 in the denominator of the expression for T_A occurs because in the ionized region there are two particles per H atom. Thus

$$\frac{p_A}{p_C} = \frac{y}{x} = \frac{2n_A T_A}{n_C T_C} = \frac{200}{\xi}.$$

ξ	y/x	x	$2u_A$	V	n_B/n_C
1	200	0.0041	14.2 km/sec	16.7 km/sec	3.94
10	20	0.049	3.3 km/sec	4.4 km/sec	3.27

$V - 2u_A$ has the values 2.5 and 1.1 km/sec, respectively; in the first case the difference is small compared with V and $2u_A$. Now our implicit assumption of the constancy of u_B in the region B requires that the conditions there resemble those in a plane slab. This is true when the shock and ionization fronts are close together, as they will be in the first

At first the H II region expands relatively fast, with a speed of about 10 km/sec, but this drops gradually. At 10 km/sec it takes about 10^5 years to ionize a sphere one parsec in radius. To judge by van de Hulst's qualitative argument and Laporte's remarks on Taylor instability it seems likely that the ionization front is unstable with respect to deformations of some kind. By breaking up the H I region into isolated zones immersed in ionized hydrogen, this instability will help the radiation to penetrate into the neutral hydrogen and will speed up the whole process.

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CHAPTER 31

PROBLEMS CONNECTED WITH THE ORIGIN OF SPIRAL ARMS

BY

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I do not intend to discuss general cosmogonical questions, but it may be worthwhile to discuss the observational data presented here as far as they refer to possible changes in the structure of what we now call the spiral arms in our own galaxy, and to compare them with our knowledge about other galaxies.

Some care is necessary in the use of the word 'spiral arm'. Its meaning may not be quite unambiguous. We may tend to call phenomena of a very different nature by the common name of a spiral arm. If this should be so, it would not be surprising because any deviation of the rotation from the uniform rotation which would correspond to the motion of a rigid body will tend to exert a shear on every structure that may be formed inside a galaxy. Hence we should expect that all individual structures which stay in existence for a sufficiently long interval of time will be more or less distorted into a shape which would look like a spiral arm or part of such an arm.

There is one feature in common to all spiral structures that have been found so far: They are always connected with interstellar matter. We see spiral structures of dust, of emission nebulosities and of those stars that belong to population I. With the possible exception of barred spirals the spiral structure does not seem to be visible at all, if we picture a galaxy in which only the population II stars are visible. This fact is not surprising. We know that the mean free path of a star with respect to gravitational interactions with single other stars is larger than the diameter of a galaxy. Hence no hydrodynamical approximation can be used to describe the behaviour of a system of stars. Structural details of star clouds will tend to disappear unless they are gravitationally stable. On the other hand systems which can be described in the hydrodynamical approximation, like gas clouds, will show the phenomena of

turbulence, compressibility and shock waves, all of which tend to produce cloudlike structures which can be distorted by differential rotation.

The observational facts which have a bearing on our problem can be considered in three different groups: those referring to barred spirals, to ordinary spirals and to our own galaxy.

In barred spirals the existence of precisely two spiral arms is most conspicuous. The arms seem to be prolongations of the bar. Looking at the pictures of barred spirals we would conclude that there is differential rotation in their outer parts which distorts the prolongation into spiral shape and which finally will leave them in the form of the only structure which is stable with respect to the shearing tendency, namely a ring. Under this point of view the ring seen in the Θ spirals would have to be considered not as an early structure but as the result of a very complete distortion.

Yet there seems to be a difficulty in this interpretation of the barred spirals. One would expect the bar to rotate too. Dynamically this seems quite plausible. We would then compare the bar with the well known prolongated ellipsoids of Jacobi used in the theory of rotating incompressible fluids. We would further have to conclude that the bar is rotating with strictly uniform velocity because it does not show any spiral distortion in itself. In some cases (or in all cases) the nucleus of the bar might rotate with a higher angular velocity than the bar itself because, as Dr. Hubble has pointed out, there are cases in which a spiral structure can be seen inside the nucleus. However, Dr. Minkowski informed me that Mayall has searched for rotation in the bars and has not been able to find signs of rotation there. I do not know what other model could be proposed that would explain the observed features of barred spirals without a rotation of the bar, thus the problem of the barred spirals probably cannot be fully understood to-day *.

In ordinary spirals we see all transitions between the case of precisely two coherent arms which may be wound around the centre several times, as in the case of M 81, and the other extreme of a great many spirally distorted lumps of matter, giving the general impression of spiral structure without any very long stretched arms. It seems difficult to

* Note added after the symposium: During the symposium I wrote a letter to Dr. Mayall about this point. In his answer he told me that he is just completing the measurement of spectrum line inclinations from which rotations can be deduced. With respect to the barred spirals he goes on saying: "What Minkowski told you is correct, but of a preliminary nature. There can be little doubt that barred spirals rotate, but evidently some of them turn so slowly that their spectrum lines are too little inclined for measurement with the Crossley nebular spectrograph."

understand how precisely two spiral arms should have been formed without the influence of a far reaching field of force, such as gravitation or perhaps a system of magnetic fields connected with a current system. Even if originated in such a way, the two spiral arms would probably not preserve their identity for a time comparable with the age of the galaxy except in a region of the system where rotation is nearly uniform. In a region of marked differential rotation spiral arms should be destroyed by the shearing effect, and the fact that they are still visible would probably necessitate the assumption that they can be formed in recent times. Qualitatively these considerations seem to be supported by the observation that in M 31 and M 33 the empirical law of rotation does not deviate appreciably from rigid body rotation in those regions of the system in which two spiral arms can be clearly distinguished on the prints available in published literature.

With respect to our own galaxy I want to stress the difficulty mentioned by Dr. Oort, with respect to the possible lifetime of the spiral arms observed by Morgan to which the spiral arms seen on the pictures which have been shown to us by Dr. Bok may be added. Even if the spiral arms should be gravitationally stable, as it is assumed in the paper by Fermi and Chandrasekhar, this stability will only refer to lateral expansion, but will not counteract the shearing effect of differential rotation efficiently. This difficulty might induce us to assume, that we are not living in a region of the galaxy in which rather stable long stretched spiral arms exist and that Morgan's arms are more or less local clouds which are spirally distorted rather than parts of arms which would wind all around the system.

On the other hand the observations in the 21-cm range reported by Dr. Oort indicate the existence of a structure winding around the centre. If such a structure should be stable for more than a few hundred million years, no other conclusion seems to be possible but that it is not a spiral but a ring of more or less circular shape surrounding the galactric centre. If this were true, Morgan's spiral arms would rather be comparable to condensations within such a ring which should be tilted with respect to the tangential direction of the ring itself. The structures observed in a distance from the galactic centre comparable to that of our sun cannot quite easily be compared to spiral arms in M 31 or M 33 because we are living in a region in which the law of rotation is definitely different from uniformity.

I should like to make a few remarks about the questions raised by Dr. Cowling in connection with the paper by Fermi and Chandrasekhar

as to the gravitational stability of spiral arms. I feel this is one of the questions which cannot be answered quite easily. The fact that Fermi and Chandrasekhar found a very plausible value of the magnetic field strength from the assumption of a gravitational stability of spiral arms does not in itself prove that arms are stable. It may also be due to the empirical fact that there is a rough equality (theoretically sometimes interpreted as an equipartition) between the energy densities of turbulence, cosmic rays and those magnetic fields which must be postulated, if we want to explain by them the interstellar polarisation and the apparent fact that cosmic rays are kept inside the galaxy. This is true because systems which form parts of a galaxy must be on the verge of gravitational stability, independently of their size, if their inner energy is comparable to the kinetic energy of their share in the rotation of the galaxy as a whole. For the sake of simplicity let us compare the galaxy to a ellipsoid of homogenous density in which the centrifugal force due to rotation is just compensated by gravitation. If we then cut out of that system a smaller ellipsoid similar in shape to the total system and if we give the smaller ellipsoid the rotation corresponding to its angular velocity in the larger system, it will just be in the same state of precise compensation of rotation and gravitation. In fact, the gravitational potential energy of a system of a given density ϱ_0 and a radius R, from which follows a total mass $M \approx \varrho_o \cdot R^3$, will be proportional to

$$U = G\, M^2/R = G\, \varrho_o{}^2 R^5.$$

On the other hand the kinetic energy of rotation in a uniformly rotating system is given by

$$T = \frac{M}{2} \cdot v^2 \sim \varrho_o\, R^5 \omega^2.$$

This means that for a given density both the kinetic and the gravitational energies are proportional to the same power of R and hence will not depend on the choice of R.

If we now ask how spiral arms can be formed, I confess that I do not see any other cause for their formation than the combined effects of turbulence, compressibility and gravitation. In order to calculate these effects we ought to have a mathematical theory of turbulence at high Mach numbers. This theory does not yet exist. But I should like to make one remark about its possible nature. Dr. Lighthill said yesterday that at high Mach numbers the sound emitted by a turbulent flow will tend to be transformed into shock waves (N-waves). I think this is true not

only for the sound but for the turbulent flow itself. Probably we should think of the irregular motions at high Mach numbers as of a statistical ensemble of shock waves. These shock waves may very well produce the density fluctuations which seem to be necessary in order to form new clouds when the earlier clouds will have been destroyed by the shearing forces. Dr. v. Hoerner will give you the results of some calculations in which the perpetual formation and destruction of spiral arms according to these views is considered.

CHAPTER 32

CONTRIBUTION TO THE TURBULENCE THEORY OF GALAXIES

BY

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I

In 1948 v. Weizsäcker [1] tried to give a hydrodynamical picture of the origin and development of galaxies.—This theory offers two possibilities for obtaining predictions about the change of areal density as a function of radius.

1. A rotating gas cloud, which is stable with respect to gravitation, contracts to a flattened disk by the decay of its inner turbulence. The radial distribution of areal density in this state may be similar to a Gaussian curve. For such a disk, which rotates in its own gravitational field, one assumes in first approximation equilibrium between gravitation and centrifugal forces, with the turbulent friction as a small disturbance. This friction has the general tendency to transfer matter towards the centre in the inner part of the nebula, and towards the outside in its outer parts. The corresponding equations have been derived and discussed by v. Weizsäcker [2], and special solutions have been given by Lüst [3] and Trefftz [4].

Wyse and Mayall [5] gave a numerical method, valid for rotational symmetry, for calculating the radial dependence of the central force (and therewith the angular velocity) on the areal density and vice versa. This method can be brought into such a form, that there is a linear relation between the values of the variables for a discrete set of radii. The matrix of the transformation (and its reciprocal) does not depend on the special problem [6].—Inserting this connection between density and angular velocity into v. Weizsäcker's formulae, one obtains a linear system of equations for the time derivative of the density under the influence of friction. One example has been treated numerically. It confirmed previous estimates. After 10^7–10^8 years, the density as a function of radius

decreases from a relatively high value at the centre first steeply and then more slowly. The distinction between the cusp at the centre and the flatter surrounding region may be quite marked.

The condensation of a considerable amount of matter into individual stars, which sets in later, ends this hydrodynamical development, so that there can be only small changes of the total system thereafter.

2. According to v. Weizsäcker [7] the spiral arms can be understood as cloud like accumulations of the dust and gas, left over after the birth of the stars. These big accumulations are created and destroyed by the turbulence coming from the differential rotation of the nebula. The differential rotation in turn is responsible for the deformation of clouds of any shape (during their lifetime) into spirals. A considerable influence of gravitation on the formation of big clouds might lead to a preference for the development of just two arms.—The early stars, whose presence is characteristic for the spiral arms, could be there newly generated or merely rejuvenated.

If this theory is correct, it should be possible to obtain the dependence of angular velocity on the radius from the shape of the spiral arms [8]. Assuming (as a first approximation) equilibrium between gravitation and centrifugal forces, this yields the distribution of mass. As parameters we need the age of the arms and the total mass of the nebula. The age of long and wound-around arms should be nearly the mean lifetime of the biggest turbulence eddies and has been estimated as 6.10^7 years, the total mass being taken as 10^{44} g. With a mean spiral shape, obtained from 5 nebulae, and with different values for the mean age, the calculation gave in all cases qualitatively the same picture as found in section 1: a cusp at the centre and a slow decay in the outer parts.

3. In order to compare the above results with purely empirical values, the radial distribution of luminosity was taken as the mean over 5 nebulae. Again we obtain a central cusp, especially pronounced, and outwards a slow decay.

We obtained qualitatively the same results by use of independent methods, and this may be regarded as an argument for the applicability of the turbulence theory. In all investigated cases the mass of the centre was estimated: it lies between 3% and 8% of the total mass of the nebula.

II

A further application of the theory was made in a paper on the spectrum of turbulence in the Orion nebula [9]. The radial velocities of

Campbell and Moore[10] were treated statistically and were found to be in good agreement with the spectral law of turbulent motion, i.e. the mean relative velocity of two points is proportional to the cube root of their distance.—A further result of this investigation was a value for the dust density in the nebula of 2.10^{-22} g/cm^3 in the central region, and 5.10^{-23} g/cm^3 for the whole nebula, in sufficient agreement with other estimates.

But it was not possible to explain the high value of the partial line-width due to turbulence. This gave a turbulence velocity for small areas which is 3 times as high as the extrapolated value from the spectral law.—With regard to the lectures of Prof. Biermann and Dr. Schlüter, we may perhaps consider this discrepancy as created by small irregular shock waves, produced by the influence of the hot stars within the nebula.

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CHAPTER 33

A MODEL FOR THE FORMATION OF GALAXIES

BY

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One might begin by considering a universe that initially is in a highly condensed state. The condition for an individual galaxy to form is that in a large region the density of matter must sufficiently exceed the average density, in order that the expansive tendency due to heat motion and other random motions may be overcome by gravitational attraction. If the universe is supposed to be initially homogeneous, there may not be sufficient time for the spontaneous appearance of large density fluctuations over extended regions. To overcome this difficulty Gamow has proposed that the universe may have been created in a highly turbulent state. Such an assumption, however, leads to other difficulties, since the properties of this primeval state of turbulence cannot be predicted without introducing a number of arbitrary further assumptions.

In contrast with Gamow's proposal, one may consider the steady state model advanced by Bondi and Gold. In this model there is a continuous creation of matter which keeps the average density of the expanding system constant, and all large scale features of the structure are invariant with time. Since we are not concerned with the formation of the first galaxy, we have the advantage that any particular galaxy is born into a universe that is already full of galaxies. Since the already existing galaxies continuously perturb the intergalactic gas, the possibility for the formation of new galaxies is always present.

It follows that we must determine that distribution of galaxies which can propagate itself. According to views developed by Hoyle [1] and McCrea [2], the expansive tendency of the gas will be overcome only for concentrations of matter of a density of at least 3ϱ, ϱ being the average density. Hence we must look for a mechanism which can systematically build up such an increase of density.

The following mechanism is proposed to this effect (see Fig. 1):

A galaxy with mass M is moving through the gas with a peculiar velocity V, or, what comes to the same, gas is streaming with the velocity V past a galaxy. Despite the expansive tendency of the gas, the paths of all particles lying within a cylinder of radius p_m of order $(M/4\pi\varrho)^{1/3}$ become convergent in consequence of the gravitational attraction of the galaxy M. These particles consequently will occupy a region extending to a distance $d = p_m{}^2/2l$ from M, where $l = GM/V^2$ (G: constant of gravitation).

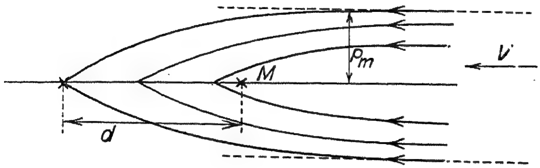

Fig. 1. Flow of intergalactic gas past a galaxy.

In parts of this region sufficiently near the axis the density will exceed 3ϱ, so that a new galaxy can form through the action of gravitational instability. This new galaxy may escape into the general field, or it may be captured by the original galaxy and so form an embryonic cluster.

We require that this mechanism should give rise to a distribution of galaxies whose average properties are independent of time. This requirement leads to several conditions which enable us to determine these properties.

First, the new galaxy must have the same mass as the old. A detailed calculation shows that this condition leads to the equation

$$V \sim 10\, G^{1/3}\, M^{1/3}\, \varrho^{1/6}.$$

With $\varrho \sim 2.10^{-28}$ g/cm^3 (the value for the steady-state universe) and $M \sim 10^{43}$ g this gives

$$V \sim 10^7 \text{ cm/sec},$$

which is of the same order as the observed peculiar velocities.

Second, the average intergalactic distance will be of order

$$d = \frac{p_m{}^2}{2l} \sim 2.10^{24} \text{ cm},$$

again in agreement with observation.

Third, the mass M can be determined from the condition for gravitational instability. The temperature of the gas can be presumed to be $\sim 10^4$ °K since its rate of cooling rises very rapidly at this temperature (I owe this remark to Mr. F. Hoyle). This leads to the value of $\sim 10^{43}$ g used above.

Fourth, each birth process takes a time of order τ (Hubble's constant). This is the time-scale required in order that receding galaxies be replaced at the correct rate.

Finally, it can be shown that this steady-state distribution of galaxies is stable, so that it is justifiable to compare it directly with the observed distribution.

We may add that other properties of the distribution e.g. the rotation of galaxies [3], the existence and separations of large clusters, the relative number of clusters of different sizes, etc. can also be obtained, but the existing observational data are not sufficient for a detailed comparison with theory [4].

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CHAPTER 34

DISCUSSION ON TURBULENCE IN GALAXIES

Chairman: Dr. T. G. COWLING

FRENKIEL: Professor George Gamow and I are looking into the problem of primeval turbulence. We are trying to describe the turbulence in the primordial gas using as basic information galaxy counts made at various observatories and more particularly by Shapley at Harvard and Shane at Lick Observatory. This work is the object of a thesis prepared under our direction by Mrs. Vera Rubin whose first results seem to be much more encouraging than we had expected (V. Rubin, Thesis for Ph. D. degree in Astronomy, Georgetown University; in preparation).

The space distribution of the galaxies is being investigated assuming that their irregular distribution is the result of the irregular density distribution of the primordial gas from which the galaxies have condensed. Regions of initial high gas density should therefore be indicated by high galaxy density. Local deviations from an average density can be expressed in terms of a fluctuating density whose mean value is zero. The statistical character of the fluctuations is described by a power spectrum or a correlation function.

We now consider a small solid angle with its apex in our galaxy and extending to successive firmaments (radial distances from our galaxy). The average mass of primordial gas contained in such a solid angle is equal to the product of its volume by the average density. As long as this volume is small compared to the scale of turbulence, the total mass contained in a sample solid angle will fluctuate around this average mass. Theoretical equations can be given relating the standard deviation of the fluctuating mass values to the standard deviation of the fluctuating local densities.

These equations are given in terms of a solid angle, a radial distance, and an assumed correlation function, and have been derived for two cases: (1) in which the absolute magnitudes of all galaxies are equal; (2) in which the absolute magnitudes of the galaxies are distributed according to an assumed function.

From the galaxy counts, one now finds the standard deviation of the fluctuating number of galaxies counted to limiting apparent magnitudes. A comparison of the observed points with the theoretically derived curves should determine the general form of the correlation function and the order of magnitude of the scale of turbulence and standard deviation of the fluctuating density of the primordial gas.

SCIAMA: There is no theoretical prediction with which the result may be compared.

GOLD: The gas lanes found by Zwicky between galaxies a few diameters apart may be interpreted as the umbilical cord in Sciama's theory.

HOYLE: In reviewing the processes that lead to the formation of a spiral arm, should we not say that the hero is the dust particle? Presence or absence of dust makes by its cooling effect a difference of a factor 100 in pressure. At the time of formation of the galaxy the temperature is probably high. During the later cooling dust particles may be formed in certain regions. The pressure then tends to be equalized by gas motions along the lines of the pressure gradient.

GOLD: Dust is a disease. It breaks out where matter is cold enough to permit the condensation of small particles. But in turn the presence of dust particles provides the most rapid agency for the cooling of gas. Gas may remain hot for long periods until it is "infected" with dust and may then cool rapidly, causing the formation of much more dust. Some of that in turn may infect other regions. The process of infection will, of course, operate most decisively along planes of shear in the medium. This may help us to understand the occurrence of the long drawn out lanes of obscuring matter.

MINKOWSKI: The suggestion made by von Weizsäcker, that smooth spiral arms occur only in the part of the nebula that has solid body rotation, is not correct. All measured velocities in M 33 refer to gas condensations in spiral arms.

VON WEIZSÄCKER: But are the arms broken or smooth? Is there a difference in appearance between the part that rotates as a solid body and the part with differential rotation?

OORT: The observations are difficult but I certainly do not see a striking difference.

MENZEL: If we regard a galaxy as a mass of ionized gas, contracting gravitationally in the presence of a uniform magnetic field, dynamo action occurs. The voltage established around the loop is of the order of 10^{16} volts. This produces a ring current that builds up slowly with the time, limited by the inductance of the system.

One can show that the electromagnetic forces tend to force such a body to assume a flattened disc shaped form. If the ring current can now separate itself from the central mass, we can expect a form similar to the Barred Spirals. Further evolution of the current bearing element could conceivably lead to the formation of spiral arms.

VON KARMAN: I have some naive questions. What is the third dimension in these pictures of bars and spirals?

VON WEIZSÄCKER: It is of the order of a few hundred parsec, i.e. one or two percent of the diameter of the disk.

VON KARMAN: So it is a hydrodynamical problem how it becomes so flat. Has this something to do with the pulling out of the Helmholtz vortex lines? If the bar, or arm, would be in rotation around its long axis, then it would have a natural tendency to be pulled out and become flat and long.

VON WEIZSÄCKER: Perhaps to these and other remarks I should clarify the meaning of the word turbulence in this connection. The compressibility changes it very much. Even inside spiral arms density differences of a factor 100 are common. In that case it is difficult to talk about vortex motion and more adequate to talk about collisions.

VON KARMAN: My imagination is not handicapped by any knowledge of the facts.

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CHAPTER 35

ON CONTRACTION IN COSMIC GASEOUS MASSES

BY

F. HOYLE

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For all hydrodynamic processes that may occur in cosmic gas masses, great importance is to be attached to sources of energy.

Oort has mentioned early-type stars as a possible driving mechanism for motions in the interstellar gas. There may be other driving mechanisms, associated notably with the processes leading to the formation of galaxies. These mechanisms might easily set up motions with speeds of 100 km/sec or more, if there were no effective dissipative mechanisms.

The following model will be discussed: Consider, as originally present, gas in a very large volume, much greater than the present volume of a galaxy, with a mean density of 10^{-27} gr/cm^3. We also consider a time scale of 10^9 years or longer. It is supposed that initially there is no dust, and no molecules.

In this mass of gas contraction may occur through gravitation. We can see that the contraction has to be almost isothermal; for, so long as the temperature is below about 10^4 K, the hydrogen is practically unionized, and there is no radiative process for dissipating the energy released by contraction, so the temperature would tend to rise; on the other hand if the temperature were high enough for an appreciable fraction of the hydrogen to be ionized, say 1.5×10^4 K, the rate of radiation by the hydrogen would exceed the rate at which energy was supplied by contraction, and the temperature would fall. It follows that the temperature must lie somewhere in the range 10^4 K to 1.5×10^4 K, which is a nearly isothermal condition.

The thermal energy that is made available by contraction is

$$-\int_{\text{isothermal}} p\, dv = M\, R\, T \log (V_i/V), \qquad \text{I}$$

where M is the mass of the cloud, T is the temperature, V_i the initial volume, V the volume after contraction, and R is the gas constant. For

comparison the gravitational energy released during the process is approximately:

$$\frac{G M^2}{V_i^{1/3}} \left[\left(\frac{V_i}{V}\right)^{1/3} - 1\right]. \qquad \text{II}$$

Now the condition that must be satisfied, in order that contraction shall take place, is that the initial gravitational potential is larger than twice the thermal kinetic energy (turbulent pressure supposed absent initially):

$$\frac{G M^2}{V_i^{1/3}} > 3\, M\, R\, T.$$

Hence in the case where contraction takes place the expression II has a larger factor multiplying the term in brackets than the factor that multiplies the logarithmic term in I.

The excess of energy released by gravitation over the thermal energy released (and radiated away) must be a source of mass motions, turbulence and shock waves. Now unless a large proportion of the energy released by gravitation is lost from the system by radiation the contraction cannot be permanent, since a permanent contraction demands a loss of energy from the system. A conversion of gravitational energy into mass motions and turbulence does not imply a permanent condensation, since a re-expansion of the condensation can occur. From what has been said concerning the expressions I and II it is clear that the most favourable case for permanent condensation is when $G\, M^2/V_i^{1/3}$ is only slightly greater than $3\, M\, R\, T$. For $T = 10^4\, K$ and a density of 10^{-27} gm per cm^3 this case gives M equal to about $3 \times 10^9\, \odot$, a typical mass for a galaxy. Even in this most favourable case, however, too great a decrease of volume leads to gravitational energy not being dissipated as radiation, but in the feeding of the mass motions that can re-expand the condensation. This can be seen from a comparison of the factor in brackets in II with the logarithmic factor in I. The factor in II becomes much larger than the factor in I when V_i/V becomes large, implying the generation of mass motions. What happens in such a situation?

As a result of the moderate degree of permanent contraction that must occur the density increases and some regions inside the cloud may themselves become unstable against condensation. Hence we must expect that there will appear:

(a) dynamical motions;

(b) formation of sub-regions.

If now we consider a sub-region that is condensing, we must expect that, just for the same reasons, sub-sub-regions inside it will become unstable, so that a hierarchy of structures may be expected (forming, so to say, an instance of "gravitational turbulence").

The characteristic time scale for condensation is $(\varrho\, G)^{-\frac{1}{2}}$. So if we had a whole series of steps in the hierarchy structure, supposing the density to increase by the same factor in each step, say by a factor 9, the time required for each step would decrease proportionately by 1/3. The sum of times for the whole hierarchy sequence would be simply

$$(\varrho_i\, G)^{-\frac{1}{2}}\left(1+\frac{1}{3}+\frac{1}{9}+\ldots\right)=\frac{3}{2}(\varrho_i\, G)^{-\frac{1}{2}}\,,$$

where ϱ_i is the initial density. So a fragmentation along a very large number of steps of such a hierarchy sequence takes only a little longer than the time for the first step.

Now what is going to put an end to such a hierarchy sequence? Evidently the contraction will cease to occur isothermally when radiation can no longer escape from the system. This will occur when the fragments become sufficiently opaque. When will this opacity be reached? It is tempting to associate the origin of Type II stars with this stage.

A word may be added about a change over from an isothermal to an adiabatic situation. Contraction in the latter case yields thermal energy given by

$$-\int_{\text{adiabatic}} p\, \mathrm{d}v = 1.5\, M\, R\, T\left[\left(\frac{V_i}{V}\right)^{2/3}-1\right], \qquad \text{III}$$

and the factor in brackets here increases more rapidly with decreasing V than the corresponding quantity in II. Hence in the adiabatic case there is no tendency towards fragmentation, except initially in the case where $G\,M/V_i^{1/3}$ is appreciably greater than $3\,M\,R\,T$.

These remarks all apply to the early stages in the formation of a galaxy and depend on the absence of dust and molecules from the gas. The situation is different when dust and molecules are present. In this case we have to deal with low temperature clouds of gas. Here there is a problem to explain why such a cloud persists, why it does not collapse under its own gravitation in a very short time. The time scale $(\varrho \mathrm{G})^{-\frac{1}{2}}$ would make us expect, with $\varrho = 10^{-21}$ gm/cm³, that condensation would occur in a few million years, yet it seems highly doubtful whether the interstellar gas clouds do all condense as rapidly as this. The following more or less speculative remarks are added on this problem.

One effect that probably works against gravitation is the pressure developed in the interstellar gas through the absorption of radiation from early type stars. Whether this is the whole story seems somewhat doubtful, however. Observation shows us rather more dense globules, of say 100 ⊙ in mass, than we should expect to find on such a basis. It is possible that the condensation of the interstellar clouds in general, and of the dense globules in particular, is a nearly adiabatic process, in consequence of the presence of molecules with high opacity in the infrared. If this is so, the whole condensation process must be radically different from that described above. Indeed condensation only occurs as a result of radiation emitted from the surface of the cloud. Now such a cause of condensation leads to a very slow shrinkage, taking much longer than the million years calculated from $(\varrho G)^{-\frac{1}{2}}$, which is only applicable under the isothermal conditions considered earlier. So a high opacity of molecules in the infra-red can protect the interstellar clouds from immediate condensation and fragmentation into stars. But given a sufficient degree of contraction, fragmentation must occur, for the reason that sooner or later the temperature inside a cloud must rise high enough for molecules to become dissociated and for dust to be evaporated. It can be shown that such a situation will arise for a globule of mass 100 ⊙ when the radius has shrunk to about 10,000 astronomical units, at which stage the internal temperature rises to several thousand degrees. The number density of atoms would then be about $10^8/\text{cm}^3$. The fragmentation of such a globule would lead to the formation of a shower of stars in a small volume, as seems to be indicated as the process of formation of Type I stars from the work of Blaauw.

Discussion

Batchelor: (1) The distinction between adiabatic and isothermal contraction seems valid only if the pressure that opposes the contraction is the kinetic pressure and not if it is the turbulent pressure. The time scale is too long for the turbulence to give rise to an appreciable kinetic pressure by means of dissipation, so that there does not seem to be any way in which the presence of strong radiation can affect the turbulent pressure.

(2) In earlier discussions we heard that gravitation was not relevant in the formation of gas clouds. How is it that now Hoyle says that it is difficult to prevent the formation of too many stars by gravitational contraction?

HOYLE: Your second question is answered by the fact that I talked about very massive clouds, about 10^9 or 10^{10} solar masses. With regard to your first question: when I spoke about mass motion I intended turbulent motion to be included.

BATCHELOR: But would not any small initial turbulence be rapidly amplified by the contraction?

GOLD: I do not see the rigid distinction between thermal and turbulent pressure in this connection.

BATCHELOR: The point I am making is that heat transfer by radiation can affect the turbulent pressure only after some of the mass motion has degenerated into thermal motion.

BONDI: But if you bring the energy into shock waves, it may be dissipated at once.

BATCHELOR: That may be true.

VON WEIZSÄCKER: Until Hoyle's theory has been extended to show how the clouds get rid of their angular momentum, I feel the estimates of the time scale etc. are not final.

HOYLE: Indeed, I have not discussed a complete theory but only the first stages. Angular momentum problems occur later, in the pre-stellar stage.

VON WEIZSÄCKER: I can see that contraction towards the galactic plane must occur, but not that the protogalaxy contracts also *in* this plane.

HOYLE: This assumption is needed in order to explain the "halo" of globular clusters and R R Lyrae stars, which extends about a factor 2 beyond the ordinary galaxy.

OORT: That factor is about correct, but it is not very big.

KANTROWITZ: Why is the appearance of gas clouds so different from that of the Galaxy if they are formed by similar processes with just a difference of scale?

HOYLE: The physical conditions operative in the gas clouds of interstellar space differ from the conditions in the primeval cloud giving rise to a galaxy in two important respects. One is that of rotation which is important in the present galactic disk, and the other is that of temperature. The temperature I took to be about 10,000° K in the primeval cloud. The temperature in the dense interstellar clouds is about 100° K.

PART VI
ACCRETION PROBLEMS

CHAPTER 36

MOTION OF STARS THROUGH CLOUDS: ACCRETION

BY

W. H. McCREA
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The following is a summary of some of the work devoted to this particular topic and is designed mainly as a guide to the literature. No reference is made to a great deal of other work that is highly relevant but is not principally concerned with the problems mentioned. *

1. Problems of steady motion

In the basic discussion a *star* is treated as an isolated massive gravitating body and (save in so far as it is disturbed by such a star) a *cloud* is treated as an unbounded uniform distribution of interstellar matter whose self-gravitation is ignored. (When required by considerations of convergence, "cut-off" effects owing to the existence of other stars can be postulated.) The interstellar material, or *medium*, is taken to consist of neutral atoms and a small proportion of dust. This means that the medium can be treated as either (P) an assembly of *particles* that can suffer inelastic collisions, or (G) a *gas* that can lose heat-energy (by radiation). Two extreme cases arise in each treatment: P_v, P_t in which the speed V of the star through the cloud is large or small compared with the mean thermal motion of the particles, and correspondingly G_v, G_t in which the star's motion is highly supersonic or highly subsonic. Following Bondi, we call P_v and G_v the *velocity-limited* case, and P_t and G_t the *temperature-limited* case. In P_t and G_t the flow is treated as spherically symmetric relative to the star.

The object is to determine the flow in the medium, the rate of accretion by the star and, in the velocity-limited case, the interaction-

* In presenting his paper at the Symposium, Dr. McCrea quoted explicitly the chief formulae in the theory and sketched their derivation. He also quoted some of his numerical results [21] regarding the possible formation of groups of O and B stars.

force between the star and the cloud, assuming that a steady state has been established.

Case P_v. Here the fundamental work is that of Hoyle and Lyttleton[12–16] who recognized the importance of inelastic collisions in the vicinity of the "accretion axis" and evaluated the consequent rate of accretion. Bondi and Hoyle[2] elaborated this work by studying the flow in the accretion stream: they found that this is not fully determined by steady-state considerations alone, though the rate of accretion must be between 1.0 and 0.5 of that calculated by Hoyle and Lyttleton. Also Bondi and Hoyle evaluated the resisting force exerted upon the star. Dodd and McCrea[7] showed later that the value of the force is effectively the same even if all the requirements for the B–H process are not fulfilled; an equivalent result had already been obtained by Agekyan[1].

The possible astrophysical significance of the processes involved had been appreciated by various earlier writers including Nölke[25–27], Moreux[24], Brown[6], Moiseyev[22, 23]. However, it seems correct to assert that a realistic description of the physical conditions established near the accretion axis, and of the effect of these conditions upon the inferred consequences, had not been obtained prior to the work of Hoyle and Lyttleton. These conditions have recently been further discussed by Lyttleton[19] and by Bondi[5].

Case P_t. Some of the theory required has been given by Eakin and McCrea[11]. It can be completed by taking account of the establishment of an "accretion envelope" analogous to the "accretion stream" in P_v.

Case G_v. With the aid of an electronic computor, Dodd[9] has made a numerical study of this problem in the case of isothermal flow. His estimates of the accretion-rate are of the same order as in P_v. His work would have to be still further elaborated in order to yield an estimate of the force.

Case G_t. This has been successfully studied by Bondi[4] for the general polytropic gas. Using his results and those in P_v, he conjectures a general approximate formula for the accretion-rate for a general value of V. His conjecture is supported by Dodd's results.

2. Non-steady state

Some features of the formation of the accretion stream when the star enters a bounded cloud have been studied by Bondi and Hoyle[2] and by Dodd[8].

3. Applications

In the context, these are to be considered, not for their general astrophysical interest, but as showing the possible significance of the processes here concerned and the problems that demand attention in this field.

a. *Applications to existing conditions in the Galaxy.* There is probably nothing that can yet be claimed as direct observational evidence for the occurrence of the phenomena concerned. However, since these phenomena are inferred so simply from general physical principles, there can be no doubt that they do occur. The question is whether they occur on a scale that produces effects of astrophysical significance. What is at present known about the physical state of interstellar matter scarcely suffices definitely to predict the effects; the procedure has to be to study various possible consequences, and, if these agree with observed effects, to examine the plausibility of the conditions required to produce them in this way.

Of such consequences, the ones susceptible of most *detailed* study are those concerned in Lyttleton's [19] theory of comets and in Bondi, Hoyle and Lyttleton's [3] theory of the solar corona; see also Hoyle [18]. These theories involve various interesting elaborations of the theory in section 1.

The *simplest* application is to explain the existence of very luminous stars by the accretion of interstellar hydrogen as originally proposed by Hoyle and Lyttleton [13]. McCrea [20] estimates that the resisting force will cause a star to be trapped before large accretion can occur so that, if it does occur, it will do so as temperature-limited accretion. He finds the conditions required to produce typical groups of O and B-stars in this way to be not implausible (Mc Crea [21]). If the stars concerned are binary, there must be consequential effects upon their orbits as indicated by Hoyle and Lyttleton and further studied by Dodd and McCrea [10].

b. *Applications to the evolution of the Galaxy.* The most far-reaching survey of the possibilities has been given by Hoyle [17]; the possible significance of accretion has also been discussed by von Weizsäcker [29].

4. Problems

Aerodynamical-temperature-limited case. A study is needed of non-steady flow. There are numerous problems concerning the possible occurrence of shock-waves or other kinds of transition between different types of flow, and of the stability of steady flows.

Aerodynamical-velocity-limited case. Further study is needed for various types of thermodynamic behaviour of the medium, in particular of the resisting force. The processes affecting a binary star immersed in the medium require further elucidation.

Effects of stellar radiation. The extensive existing work upon the effects of stellar radiation on the behaviour of interstellar matter has still to be related to the present topics. In particular, if a star undergoes large accretion, the heating effect of its increasing radiation probably sets a limit to the process. There may be important dynamical effects of radiation pressure (Agekyan [1]).

General. The effects of the detailed structure and internal motions of the medium, and the extent to which these may themselves be affected by the processes concerned, should be considered. Also the gravitational effects of the medium may have to be more fully taken into account in studying the motion of a star *through* a cloud (this being a different problem from the one dealt with by Spitzer and Schwarzschild [28]). Finally, it is probably not yet understood how a cloud containing dust normally avoids collapse under its own gravitation; were this known, the problem would arise as to whether the entry of stars can so modify the gravitational field as occasionally to induce such a collapse.

If the present topics can be successfully dealt with, the most important astrophysical problem to which a solution could then be expected is probably that of the origin of Baade's two stellar populations.

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Discussion

KANTROWITZ: The description of the accretion process made it appear that the star would be accelerated, rather than retarded, because the accreted material would strike the back ("downstream") side of the star.

McCREA: Since the material concerned is captured by the star, the particular feature mentioned by Kantrowitz would not produce an acceleration. What matters is the net transfer of the momentum of this material, and the effect of its capture is, of course, to produce a retardation. This is taken into account by the theory. However, the main contribution to the retarding forces comes, in general, from the part of the cloud material that is not captured but whose motion is altered by the gravitational action of the star.

BOK: What you have assumed is a terrifically dense interstellar cloud. I would estimate that only inside a globule or in a very dense diffuse nebula one reaches densities of the order of 10^{-20} to 10^{-21} g/cm^3. In the large dark cloud complexes, or in dark clouds like the Southern Coal Sack, the average densities are not likely to be much in excess of 10^{-23} g/cm^3.

McCREA: In a cloud or cloud-complex of given total mass, the total number of stars that can undergo large accretion according to the theory does not depend very sensitively upon the density. I quoted merely one set of values by way of illustration.

But I must emphasize that the theory should, and in fact does, offer a

possible explanation of the production of massive stars only in rather extreme conditions. For such stars are only a minute fraction of the stellar population.

OORT: One aspect, that McCrea has not mentioned, worries me. It is angular momentum. Even if we disregard the angular momentum that is present systematically as a consequence of galactic rotation, the angular momentum that is accidentally present in a large volume should be very big—big enough to make accretion into a single star impossible.

BONDI: This problem will be discussed in my communication.

HAYES: How large is the mean free path compared with the tunnel radius? You mentioned the two extreme cases: (a) hydrodynamical case and (b) the Hoyle-Lyttleton theory. It seems to me that the question of the mean free path is something prior to this distinction, for collisions are needed to get accretion in either case. Are there enough collisions?

McCREA: This question has been fully considered by the originators of the theory and there seems to be no difficulty.

BIERMANN: Yesterday we discussed expanding H II clouds. Now it can be shown that under normal circumstances accretion cannot occur in such a cloud, since the interaction between the gas particles moving to the surface of the star and the expanding gaseous cloud is by far too strong. Only one of the two processes can therefore take place at one time. Now the energy available for the expansion is normally much greater than that which might be gained by accretion, hence those stars, which cause the expansion by their ultraviolet radiation (via heat energy and radiation pressure) cannot grow by accretion. An early type star accidentally trapped in the cloud would therefore under normal circumstances not grow at all; but perhaps a dwarf star may grow a little.

What I just said, may perhaps also be expressed as follows: As far as the arguments under discussion go, there does seem to be no objection against the usual hypothesis that a star may be "born" in a cloud (by contraction of an initial condensation), but it seems unlikely that an early type star, after having "burned" most of its available hydrogen content, can be "rejuvenated" by accretion of interstellar hydrogen (which, in case it could happen, would give it another life span as an early type star).

NOTE BY McCREA: I was not suggesting that already massive stars are rejuvenated by accretion; I was considering the possible formation of massive stars from stars initially of, say, one or two solar masses. If such a star "may grow a little" then it is easy for it to grow a lot, until, in fact, accretion is halted by some such process as that contemplated

by Biermann when the heating effect of the star becomes sufficiently effective.

BOK: Are inelastic collisions likely?

SCHATZMAN: This point is answered in my paper.

SAVEDOFF: What is the fundamental difference between symmetrical accretion and gravitational contraction?

BIERMANN: In the first case a star is supposed to pre-exist; it radiates already and this radiation may impede accretion. In the second case a mass concentration is formed, but it is not yet hot enough to liberate nuclear energy. In this case a star is born, but the luminosity comes later than the mass.

CHAPTER 37

ÉTUDE PHYSIQUE DE L'ACCRÉTION

BY

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1) L'hypothèse de l'accrétion *sous sa première forme* a été faite par Eddington [1]. Toutefois, Eddington supposait que seuls étaient captés les atomes qui frappaient la surface de l'étoile.

Cette théorie a été reprise par Hoyle et Lyttleton et par Hoyle [2], cette fois ci en supposant que tous les atomes de moment angulaire suffisamment petit finissaient par tomber sur l'étoile. Dans cette hypothèse, l'ordre de grandeur de l'accrétion est

$$A \leqq cte \frac{G^2 M^2}{v^3} \varrho_\infty , \tag{1}$$

où la constante est une nombre de quelques unités au plus et v la vitesse de l'étoile par rapport à la matière interstellaire. — Cette formule suppose essentiellement que la matière interstellaire ne s'échauffe pas lors de sa chute vers l'étoile.

Lorsque la vitesse de l'étoile par rapport à la matière interstellaire devient petite, la formule (1) n'est plus applicable et Bondi [3], étudiant l'accrétion de symmétrie sphérique autour d'une étoile au repos, a montré que, selon toute vraisemblance, il fallait dans la formule (1) remplacer la vitesse v par la vitesse de propagation du son dans le milieu.

Le cas de l'étoile immobile dans la matière interstellaire est évidemment le cas le plus favorable à l'accrétion.

2) Accrétion autour d'une étoile au repos

Bondi [3] a étudié le cas de l'accrétion de symmétrie sphérique, en supposant une relation de la forme $p \sim \varrho^\gamma$ existant entre la pression et la densité au cours de la chute de la matière. Cependant, malgré l'énorme quantité d'énergie disponible par gramme de matière au cours de sa chute vers l'étoile, si grande que l'on vient à se demander comment la matière

peut se débarasser (par rayonnement ou de toute autre manière) de cette énergie, Bondi n'a pas étudié les problèmes du rayonnement. Les présents calculs ne sont qu'une étape vers la solution de ces problèmes.

On suppose une étoile au repos, dont la température et la densité à l'infini sont T_∞ et ϱ_∞.

On néglige dans ce calcul le potentiel gravifique du gaz pour ne tenir compte que du potentiel gravifique $\frac{GM}{r}$ de l'étoile.

On étudie uniquement ici le *régime permanent.*

Si la compressibilité adiabatique du gaz, γ, peut être supposée constante, on a, par les équations du mouvement, de conservation de la matière et de conservation de l'énergie:

$$u\frac{\partial u}{\partial r} = -\frac{GM}{r^2} - \frac{1}{\varrho}\frac{\partial p}{\partial r} \tag{2}$$

$$\frac{\partial}{\partial r} r^2 \varrho u = 0 \tag{3}$$

$$u\frac{\partial p}{\partial r} = \frac{\gamma p}{\varrho} u \frac{\partial \varrho}{\partial r} - (\gamma - 1)\,\mathrm{div}\, F, \tag{4}$$

où F est le flux du rayonnement.

Le premier problème à résoudre pour savoir quelle est la nature du mouvement du gaz (adiabatique ou autre) est de savoir quelle est la valeur du terme div F.

3) Problèmes du rayonnement

Une description générale du phénomène est nécessaire avant que le calcul en soit abordé.

On suppose, en premier lieu, que le gaz interstellaire à l'infini est constitué principalement d'hydrogène neutre, ou même combiné à l'état de molécule. Dans sa chute vers l'étoile, il s'échauffe modérément, probablement de façon quasi adiabatique, en raison de son faible pouvoir émissif (bien inférieur à celui du corps noir à la même température). A une certaine distance r de l'étoile se produit l'ionisation de l'hydrogène par le rayonnement en provenance de l'étoile, ou en provenance de la matière captée et chauffée par sa chute. Là commence une zône d'hydrogène ionisé, H II, approximativement isotherme tant que les termes de production d'énergie mécanique sont négligeables et que le rayonnement est essentiellement dû aux ions présents dans le gaz. Par suite de l'augmentation de densité, il arrive un moment où l'émission continue devient notable, l'abaissement de température qui devrait en résulter étant en réalité compensé par l'apport continu d'énergie mécanique.

A partir de là, la température et la densité augmentent régulièrement jusqu'au voisinage même de la surface de l'étoile.

4) Région H I

Nous admettons ici que tout le rayonnement ultraviolet a été arrêté. Seul parvient dans cette région le rayonnement visible et infrarouge.

Le gain d'énergie dans cette région a les origines suivantes:
gain d'énergie mécanique;
gain d'énergie par photo-ionisation de l'ion negatif H^-; [4]
gain d'énergie par photo-ionisation du carbone neutre, pour former du carbone ionisé C II [5].

Les pertes d'énergie sont dues pour l'essentiel à l'excitation des molécules d'hydrogène avec émission ensuite du spectre de rotation-vibration [6].

Pour fixer les idées nous comparons les differents termes entre eux:

On a

$$u = -\frac{\lambda G^2 M^2}{a^3} \frac{\varrho_\infty}{\varrho} \frac{1}{r^2},$$

où λ est une constante de l'ordre de l'unité [3];

$$\frac{\partial p}{\partial r} \backsimeq -\frac{G\,M}{r^2}\varrho,$$

d'où

$$u\frac{\partial p}{\partial r} \backsimeq \lambda\frac{G^3 M^3}{a^3\, r^4}\varrho_\infty,$$

ou, en unités de masse et rayon solaires:

$$u\frac{\partial p}{\partial r} \backsimeq \frac{10^{-0.76}\lambda\, M^{*3}}{T^{3/2} r^{*4}} N_{H\infty}, \tag{5}$$

où $N_{H\infty}$ est le nombre d'atomes d'hydrogène par cm³ à l'infini.

La perte d'énergie par rayonnement est:

$$n_H\, L_{Hm} = n_m\, \gamma_1, \tag{6}$$

et γ_1, d'après Spitzer, est

$$\gamma_1 \backsimeq 8.62 \cdot 10^{-24 - 0.0641\,\Theta}$$

pour $\Theta > 50$ (suivant l'usage, on a designé par Θ la quantité $5040/T$).

Pour fixer les idées, à $T = 100°$, où $\Theta \backsimeq 50$, l'apport d'énergie mécanique (5) est plus grand que la perte d'énergie par rayonnement (6)

lorsque $r < 1$ parsec environ. Si l'accrétion est moindre, c.à.d. si λ n'a pas sa valeur maximum, la limite inférieure varie comme $\lambda^{1/4}$, c.à.d. très lentement. Il en résulte immédiatement, en raison de la présence d'un terme d'absorption, que la chute du gaz interstellaire dans la région H I est pratiquement adiabatique.

5) Apparition de la région H II

Strömgren [7] a calculé les conditions d'apparition des zônes dites H I et H II.

Nous supposerons la présence d'un rayonnement ionisant en provenance des régions centrales de la zône d'accrétion, et nous calculerons *les conditions* d'apparition d'une zône H II.

Le calcul de Strömgren doit être partiellement repris pour les raisons suivantes: 1) En raison du régime stationnaire les équations de conservation et de continuité doivent être satisfaites au passage de la zône H I à la zône H II; 2) La température du gaz d'électrons dans la région H II, en raison de la présence des ions de différents atomes, peut être appréciablement différente de la température du rayonnement ionisant.

Dans la zône de transition de H I à H II, supposée très mince, on peut supposer la pression constante, et admettre que la pression totale est la somme des pressions partielles de l'hydrogène neutre et de l'hydrogène ionisé:

$$P = 2\,N\,x\,k\,T_{\mathrm{II}} + N\,(1 - x)\,k\,T_{\mathrm{I}}, \tag{8}$$

où T_{II} et T_{I} sont les températures cinétiques dans les zônes H II et H I. On admettra que la température du rayonnement ionisant T_c est différente de la température des électrons dans la zône H II.

Dans ces conditions, en posant comme Strömgren

$$c_1 = 10^{-0.51 - \Theta_c \chi}\,\frac{2q_+}{q_H}\sqrt{\frac{T_{\mathrm{II}}}{T_c}}\;T_c^{3/2}\cdot R^2 \tag{9}$$

$$\Theta_c = \frac{5040}{T_c} \tag{10}$$

$$w = \frac{R^2}{4s^2} \tag{11}$$

on a

$$R^2\,T_c^4 = 10^{15.0268}\,L_i, \tag{12}$$

où w est le facteur de dilution, L_i l'énergie totale par seconde du rayon-

nement ionisant, T_c la température de couleur du rayonnement ionisant, s la distance à l'étoile, les unités étant pour R le rayon solaire, pour s le parsec $3{,}08 \cdot 10^{18}$ cm, pour L_i le rayonnement solaire.

En posant

$$d\tau_u = (1 - x)\, N\, a_u \cdot 3{,}08 \cdot 10^{18}\, ds, \tag{13}$$

on a

$$\frac{x^2}{1-x}\, \frac{2N\, k\, T_{II}}{(1-x)\, k\, T_I + 2\, x\, k\, T_{II}} = \frac{c_1}{s^2}\, e^{-\tau_u}\,. \tag{14}$$

En posant, de la même façon que Strömgren

$$dz = \frac{N_{II}^2}{c_1}\, 3{,}08 \cdot 10^{18}\, a_u\, s^2\, ds \tag{15}$$

$$y = e^{-\tau_u}, \tag{16}$$

on obtient les équations

$$\frac{dy}{dz} = -\frac{(2\, k\, T_{II})^2\, x^2}{\{(1-x)\, k\, T_I + 2\, x\, k\, T_{II}\}^2} \tag{17}$$

$$z = \frac{s^3}{3}\, \frac{N_{II}^2}{c_1}\, 3{,}08 \cdot 10^{18}\, a_u \tag{18}$$

$$\frac{1-x}{x^2}\, \frac{(1-x)\, k\, T_I + 2\, x\, k\, T_{II}}{2\, x\, k\, T_{II}} = \frac{a\, z^{1/3}}{y}, \tag{19}$$

avec

$$a = \left\{\frac{9}{c_1\, N_{II}\, (3{,}08\, .\, 10^{18}\, a_u)^2}\right\}^{1/3}; \tag{20}$$

et l'on déduit, comme Strömgren, que la limite de séparation entre les zônes H I et H II se produit lorsque $z \cong 1$, c.à.d. lorsque

$$s_0 = \left\{\frac{3\, c_1}{N_{II}^2\, .\, 3{,}08\, .\, 10^{18}\, a_u}\right\}^{1/3}. \tag{21}$$

Numériquement, avec $\chi = 13{,}53\; eV$, $a_u = 6{,}3 \cdot 10^{-18}\; \mathrm{cm}^{-2}$, on obtient:

$$\log s_0 = 4.65 - 4.51\, \Theta_c - \frac{5}{6} \log T_c + \frac{1}{6} \log \frac{T_{II}}{T_c} + \frac{1}{3} \log L_i - \\ - \frac{2}{3} \log N_{II}. \tag{22}$$

Si l'on pose, comme Bondi:

$$x = \frac{2\Re T_{\mathrm{II}}\, r}{G\, M\, \mu_{\mathrm{II}}}, \tag{23}$$

on a:

$$\log x_0 = 5{,}54 - 4{,}51\, \Theta_c - \log T_c + \tfrac{7}{6} \log T_{\mathrm{II}} + \tfrac{1}{3} \log L_i -$$

$$- \tfrac{2}{3} \log N_{\mathrm{II}} - \log M^{\star}, \tag{24}$$

où $M^{\star}$ est en unités de masse solaire, L_i en unités d'éclat solaire, et N_{II} est le nombre d'atomes d'hydrogène par centimètre cube.

Pour trouver la valeur de la grandeur essentielle T_c qui intervient dans l'équation (24), il est nécessaire de déterminer dans la zône H II les conditions d'émission du rayonnement.

6) Structure de la zône H II

D'après Spitzer [4, 6, 8] on peut écrire pour l'équation d'énergie

$$n_e \sum_j G_{ej} = n_e \sum_j L_{ej} \tag{25}$$

dans la zône H II. Les termes principaux dans cette équation étant les termes de gain d'énergie G_{ep} et les termes de perte L_{ei}, L_{ep}, de gain par absorption des électrons dans le champ des protons, et de perte par rayonnement des ions atomiques et des électrons dans le champ des protons.

L'équation exacte d'énergie est en realité:

$$\frac{u}{\gamma - 1}\left(\frac{\partial p}{\partial r} - \frac{\gamma p}{\varrho}\,\frac{\partial \varrho}{\partial r}\right) = -\left(n_e\, L_{ei} + n_p\, L_{ep} - n_e\, G_{ep}\right). \tag{26}$$

Tant que le premier membre est négligeable devant les termes du second membre, la zône est pratiquement isotherme.

Examinons pour cela les ordres de grandeur des différents termes. A partir des formules de Strömgren et de Cillié [9], on trouve pour $n\, L_{ei}$ les valeurs suivantes lorsque $\Theta = 1$ (Tableau I):

TABLEAU I

$\log n_e$	$\log n_i$	$n_e L_{ei}$	$n_e L_{ep}$
4	1	$2.9 \cdot 10^{-15}$	$6,2 \cdot 10^{-16}$
5	2	$7.45 \cdot 10^{-14}$	$6,2 \cdot 10^{-14}$
6	3	$9.23 \cdot 10^{-13}$	$6,2 \cdot 10^{-12}$
7	4	$9.23 \cdot 10^{-12}$	$6,2 \cdot 10^{-10}$

D'autre part, le premier membre vaut

$$\frac{u}{\gamma - 1} \frac{\partial p}{\partial r} \backsimeq \frac{1}{\gamma - 1} \frac{\lambda G^3 M^3 m_H N_\infty}{(2 \Re T)^{3/2} r^4} = \frac{\lambda N_\infty}{r^{\star 4}} M^{\star 3} 10^{-6,66}.$$

Le premier membre est surement plus petit que le second lorsque r est plus grand qu'une certaine valeur r_1 donnée dans le Tableau II:

TABLEAU II

$\log n_e$	$\log r^\star_1$	r_1 en *u.a.*
4	4.3	100
5	3.6	20
6	3	5
7	2.2	1

Plus vers l'intérieur, au contraire, l'émission $n_p L_{ep}$ devient largement préponderante, et l'on a:

$$\frac{u}{\gamma - 1} \left(\frac{\partial p}{\partial r} - \frac{\gamma p}{\varrho} \frac{\partial \varrho}{\partial r} \right) \backsimeq - n_p L_{ep}.$$

Une solution approchée de cette équation est

$$T = \frac{G M}{4 \Re r},$$

ou, numériquement

$$T = 6.10^6 \frac{M^\star}{r^\star}.$$

Au voisinage de la surface de l'étoile, la température s'est considérablement élevée.

Nous supposerons donc, dans la suite du calcul, que la température de

couleur T_c du rayonnement est 10^6 degrés. La température des électrons T_{II} sera prise égale à 10000 degrés.

7) Accrétion dans la zône H II

Soient u_0 la vitesse de la matière à la limite entre les zônes H I et H II, r_0 le rayon de la sphère séparant les deux zônes. Dans l'état stationnaire on a simplement:

$$\frac{1}{2}\,u_{II}^2 - \frac{1}{2}\,u_{0II}^2 = \frac{GM}{r_{II}} - \frac{GM}{r_0} - \frac{\Re T_{II}}{\mu}\log\frac{\varrho_{II}}{\varrho_{0II}} \tag{27}$$

$$r_{II}^2\,\varrho_{II}\,u_{II} = a. \tag{28}$$

Si l'on élimine ϱ_{II} entre les équations (27) et (28), on peut écrire:

$$\frac{1}{2}\,u_{II}^2 - \frac{\Re T_{II}}{\mu}\log u_{II} = \tfrac{1}{2}\,u_{0II}^2 + \frac{GM}{r_{II}} - \frac{GM}{r_0} + \frac{\Re T_{II}}{\mu}\log\frac{r^2}{r_0^2} - $$

$$- \frac{\Re T}{\mu}\log u_{0II}. \tag{29}$$

L'étude de l'équation (29), faite à la manière de Bondi [3], montre que le taux maximum d'accrétion est atteint lorsque le maximum de vitesse u_{II} devient égal à la vitesse de propagation d'une perturbation, c.à.d. lorsque

$$u_{II} = \frac{\Re T_{II}}{\mu_{II}}, \tag{30}$$

ceci se produisant lorsque

$$r_m = \frac{L\,M\mu_{II}}{2\,\Re T_{II}}. \tag{31}$$

Lorsque ces conditions sont réalisées, il existe une relation entre u_{0II} et r_0. Si l'on pose avec Bondi

$$y^2 = \frac{u_{0II}^2\,\mu_{II}}{\Re T_{II}} \tag{32}$$

$$x = \frac{2\Re T_{II}\,r_0}{G\,M\,\mu_{II}}, \tag{33}$$

on a immédiatement la relation

$$4\log x + \frac{4}{x} = 3 + y^2 - \log y^2. \tag{33}$$

Le tableau II donne y et $x^2 y$ en fonction de x:

TABLEAU III

x	y	$x^2 y$
1	1	1
2	0.45	1.8
3	0.26	2.3
4	0.17	2.7
5	0.12	3.0
6	0.09	3.24
7	0.07	3.43
10	0.037	3.70
∞	0	4.48

Le taux d'accrétion maximum dans la zône H II est alors

$$A = \pi x^2 y \frac{G^2 M^2}{\left(\frac{\Re T_{II}}{\mu_{II}}\right)^{3/2}} \varrho_{0II}. \tag{34}$$

8) Accrétion dans la zône H I

Nous étudions tout d'abord les conditions de raccordement, de part et d'autre de la surface de séparation de rayon r_0.

Les équations de continuité de la pression et de conservation de la matière s'écrivent

$$\frac{\varrho_{0II}\, T_{0II}}{\mu_{II}} = \frac{\varrho_{0I}\, T_{0I}}{\mu_I} \tag{35}$$

$$u_{0I} = u_{0II} \frac{T_{0I}}{T_{0II}} \frac{\mu_{II}}{\mu_I}. \tag{36}$$

Nous posons

$$\frac{T_{0I}}{T_{0II}} \frac{\mu_{II}}{\mu_I} = \xi.$$

Le taux d'accrétion maximum (34) est alors

$$A = \pi x^2 y \frac{G^2 M^2}{\left(\frac{\Re T_I}{\mu_I}\right)^{3/2}} \xi^{5/2} \varrho_{0I}. \tag{37}$$

Nous calculons ϱ_{0I} en supposant, comme il a été dit plus haut, que la

chute du gaz interstellaire vers l'étoile se fasse de façon adiabatique. On a alors

$$\varrho_{0\mathrm{I}} = \varrho_{\infty\,\mathrm{I}} \left[1 + \frac{\gamma - 1}{\gamma} \left(\tfrac{1}{2}\, y^2 \xi - \frac{2}{x\,\xi}\right)\right]^{-\frac{1}{\gamma-1}}. \tag{38}$$

Le taux d'accrétion est *inférieur* au taux d'accrétion dans une région H I seul si

$$x^2 y\, \xi^{5/2} \left[1 + \frac{\gamma - 1}{\gamma} \left(\tfrac{1}{2}\, y^2\, \xi - \frac{2}{x\,\xi}\right)\right]^{-\frac{1}{\gamma-1}} \leqslant e^{3/2}. \tag{39}$$

Cette équation est satisfaite pour $\xi > \xi''$. Le tableau IV donne la racine ξ'' en fonction de x pour $\gamma = 1$ et $\gamma = 5/3$:

TABLEAU IV

x	$\xi''\ (\gamma = 1)$	$\xi''\ (\gamma = 5/3)$
1	1	
2	0.2013	
3	0.1074	
5	0.05123	
10	0.02006	0.0812
20	0.00827	
50	0.002698	0.016

Pour x grand, on a:

dans le cas isotherme, $\xi \cong \dfrac{0{,}8}{x \log x/2}$

dans le cas adiabatique, $\xi \cong \dfrac{0{,}8}{x}$.

On voit que si le rayon de la surface de séparation entre les zônes H I et H II est assez grand, le taux d'accrétion peut être *très inférieur* à celui calculé par Bondi.

9) Condition d'apparition de la zône H II (étoile faible)

Reportons dans l'équation (24) la valeur de N_{II}, telle qu'elle résulte des équations (35) et (38):

$$N_{\mathrm{II}} = N_{\infty}\, \xi \left[1 + \frac{\gamma - 1}{\gamma} \left(\tfrac{1}{2}\, y^2\, \xi - \frac{2}{x\xi}\right)\right]^{-\frac{1}{\gamma-1}}. \tag{40}$$

On obtient une équation en x et ξ (pour $\gamma = 5/3$):

$$\log x - \log \left[1 + 0{,}4\left(\tfrac{1}{2}y^2 \xi - \frac{2}{x\xi}\right)\right] + \frac{2}{3}\log \xi =$$

$$5.54 - 4.51\, v_e - \log T_e + \frac{7}{6}\log T_{\mathrm{II}} + \frac{1}{3}\log L_i -$$

$$- \log M^\star - \frac{2}{3}\log N_\infty . \tag{41}$$

Les seules solutions physiquement importantes sont celles pour lesquelles x est grand et ξ petit. Le premier membre admettant un *minimum*, on trouve des conditions limites à l'existence de la zône H II. En effet, pour ξ donné (et en négligeant y^2), le premier membre admet un minimum pour $x\xi = 1.6$. Le premier membre vaut alors $1.162 - \frac{1}{3}\log \xi$. En se rapportant à ξ_∞, on a de même un minimum pour $x\xi_\infty = 1.6/3$, minimum valant $1.144 - \frac{1}{3}\log \xi_\infty$. L'équation en x n'admet alors une solution que si:

$$1.146 - \frac{1}{3}\log \xi_\infty < 5.54 - 4.51\,\Theta_e - \log T_e + \frac{7}{6}\log T_{\mathrm{II}} +$$

$$+ \frac{1}{3}\log L_i - \log M^\star - \frac{2}{3}\log N_\infty . \tag{42}$$

Le taux d'accrétion étant donné par (37), on a:

$$L_i = \frac{G\,M}{R}\,A\,, \tag{43}$$

ou, en se rapportant à ξ_∞ et N_∞:

$$L_i = \pi\, x^2\, y\,\frac{G^3 M^3}{R}\;\frac{N_\infty\; m_H}{\left(\dfrac{\Re T_\infty}{\mu_{\mathrm{I}}}\right)^{3/2}}\cdot\left(\frac{x}{x\,\xi_\infty + 0.8}\right)^{3/2}.$$

On a $x\,\xi_\infty = 1{,}6$. En reportant cette expression dans l'équation (42) on obtient:

$$\log N_\infty = 13.188 - 13.53\,\Theta_e - 3\log T_e + \frac{7}{2}\log T_{\mathrm{II}} +$$

$$+ \log \frac{\pi\, x^2\, y\, G^3\, m_H\, M_\odot^3}{R\left(\dfrac{\Re T_\infty}{\mu_{\mathrm{I}}}\right)^{3/2} L_\odot} + \frac{7}{2}\log \xi_\infty + \frac{5}{2}\log\frac{3}{2}\,, \tag{44}$$

ou encore

$$\log N_\infty = 13.066 - 13.53\,\Theta_c - 3\log T_c + \tfrac{7}{2}\log\frac{\mu_{II}\,T_\infty}{\mu_I} -$$

$$- \log R^\star\, T_\infty^{\ 3/2}\,. \tag{45}$$

On reconnaît immédiatement, étant donné les valeurs que nous avons indiquées pour T_c et T_∞, que l'équation (45) admet une solution $\log N_\infty \lesseqgtr 0$.

Pour toute densité plus grande, la zône H II n'apparaît pas (en tout cas pas sous l'influence du rayonnement ionisant). Dans ce cas le taux d'accrétion est bien donné par la formule de Bondi pour une zône adiabatique

$$A = \frac{10^{16.995}\cdot N_\infty}{T_\infty^{\ 3/2}}\, M^{\star 2}.$$

La période du phénomène est alors (pour $T_\infty = 100°$ K)

$$M^\star \frac{dt}{dM^\star} = \frac{10^{3.80}}{M^\star N_\infty} \quad \text{milliards d'années.} \tag{46}$$

A moins de posséder une densité extraordinairement élevée (auquel cas on ne peut pas vraiment parler d'accrétion, mais plutôt de formation d'une étoile), aucun nuage de matière interstellaire ne peut subsister assez longtemps pour que l'accrétion soit possible.

10) Apparition de la zône H II (étoiles brillantes)

Dans ce cas, T_c est la température de couleur de l'étoile, L_i son éclat. L'inégalité (42) s'écrit:

$$\log \mathrm{N}_\infty \leqslant 6.6 + \tfrac{7}{4}\log T_{II} + \tfrac{1}{2}\log L_i - \tfrac{3}{2}\log M^\star -$$

$$- \tfrac{3}{2}\log T_c - 6.75\,\Theta_c + \tfrac{1}{2}\log \xi_\infty\,. \tag{47}$$

Le tableau V donne pour différents types d'étoiles la valeur limite de $\log N_\infty$:

TABLEAU V

Type Spectral	lim. $\log N_\infty$
O	6
B	4.7
A	3.5

Il est donc clair que pour les *étoiles brillantes*, le taux d'accrétion peut tomber bien au dessous du taux calculé par Bondi, à moins que la densité de la matière interstellaire ne soit extrêmement élevée.

Pour fixer les idées, nous allons calculer le taux d'accrétion pour une étoile O, lorsque $N_\infty = 10^4$. On a alors $x_0 \cong 10^7$, $x^2 y \cong e^{3/2}$ et le taux d'accrétion est

$$A = \frac{10^{16.995} N_\infty \ M^{\star 2}}{T_\infty{}^{3/2}} \xi_\infty{}^{5/2}$$

avec $\xi_\infty{}^{5/2} \cong 10^{-5.75}$, ou, numériquement

$$A \cong 10^{14.25}.$$

D'où l'échelle de temps du phénomène:

$$M^\star \frac{\mathrm{d}t}{\mathrm{d}M^\star} = 10^{3.5}\ 10^9 \text{ ans}\ .$$

Pour une étoile brillante, le taux d'accrétion peut être, en définitive, beaucoup plus petit que pour une étoile faible.

11) Les problèmes de l'accrétion autour d'une étoile en mouvement impliquent également la prise en considération des effets de rayonnement et de dissipation de l'énergie gagnée par la matière tombant sur l'étoile. Ils feront l'objet d'un travail ultérieur.

Conclusion

12) Il semble résulter des calculs présentés ici que le taux d'accrétion utilisé par Bondi et Hoyle est largement surestimé. L'importance de l'accrétion en astrophysique s'en trouverait donc considérablement réduite.

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CHAPTER 38

GENERAL DISCUSSION ON ACCRETION PROCESSES

Chairman: Dr. P. LEDOUX

HOYLE: In a discussion of the accretion process a careful distinction is needed between the following two cases:

(1) the effects produced when an early type star moves into a gas cloud;

(2) the development of a dwarf late type star into an early type bright star.

Let us suppose for the purpose of a comparison of these two cases that the velocity of the star relative to the gas is the same in both cases, and that the surface temperature and luminosity of the early type star is the same in both cases. At first sight it might seem as if the question of whether the pressure developed in the gas due to the absorption of radiation from the star can prevent accretion occurring, must be the same in the two cases. But this is not so, because the distribution of the gas is different in these cases. In case (1) the star, when it enters the cloud, is simply surrounded by gas at the normal density of the cloud. In case (2), with accretion already occurring, we have a high density of gas near the star due to the infall process, and unless the star can blow this shield of high density gas away it cannot get to work on the more distant gas of the cloud. Expressed more precisely, the radius of the H II region around the star is very different, being much smaller in case (2). The accretion criterion takes the same form in the two cases, namely

$$G M/r_{H\,II} > R\,T\,,$$

where M is the mass of the star, $r_{H\,II}$ is the radius of the H II region, and R is the gas constant. Because of the difference in the value of $r_{H\,II}$ the application of this condition leads to different results in the two cases. Care must I think be taken to distinguish between an early type star being able to continue with accretion, *once* accretion has begun (as

in case (2)), and the case of setting up the accretion process, which certainly is difficult in case (1).

BONDI: First of all, I want to stress that the accretion theory considers not only O- and B-stars, but also other smaller ones which form the bulk of the stellar population. Secondly, I want to examine the importance of direct radiation pressure. Consider the equations of motion of matter in a cone of small solid angle ω, in the neighborhood of a star of mass M and luminosity L (Fig. 1). The outward momentum gained per unit time by the matter in the solid angle, in the space between spherical surfaces of radii r_1, r_2, is given by:

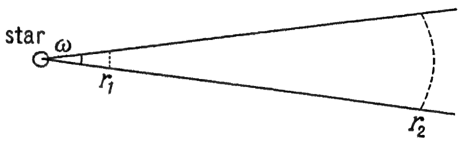

Fig. 1.

$$\frac{L\omega}{4\pi c} - \int_{r_1}^{r_2} \omega r^2 \varrho \,\mathrm{d}r \frac{GM}{r^2}.$$

Here it has been assumed that all radiation is absorbed so that the effect of radiation pressure is overestimated. The solid angle is of no importance. It is seen that attraction is preponderant if:

$$\frac{L}{4\pi c} < \int_{r_1}^{r_2} \mathrm{d}r\, \varrho\, G\, M,$$

which can also be written:

$$\frac{L}{4\pi c G M \varrho_\infty} < \int_{r_1}^{r_2} \frac{\varrho}{\varrho_\infty} \,\mathrm{d}r,$$

or introducing convenient units:

$$\frac{L}{L_\odot} \frac{M_\odot}{M} \frac{1}{n} \times 15 \text{ parsec} < \int_{r_1}^{r_2} \frac{\varrho}{\varrho_\infty} \,\mathrm{d}r$$

(where n is the number of hydrogen atoms per cm^3 corresponding to ϱ_∞).

In the case of a heavily accreting star we may put $r_2 = 10 - 30$ parsec, while r_1 is very much smaller; further ϱ will be equal to or larger than ϱ_∞. Therefore the integral has a value of at least 15 parsec. The condition then becomes:

$$n > \frac{L}{L_\odot} \frac{M_\odot}{M}.$$

For very massive stars this may give $n > 10^3$, for smaller stars a lower figure will result.

Near the star the density ϱ will greatly exceed ϱ_∞ if the star is already accreting; this will make the situation even more favorable for further accretion.

In reply to Schatzman, I would observe that it follows from McCrea's work that the cooling process apparently is of little influence; the retarding force is independent of it; the stopping time will not be affected, and once the star is stopped, it will be of very little importance whether the process is isothermal or adiabatic.

In reply to Oort, concerning the angular momentum: the process of accretion is a mechanism to select favorable regions where angular momentum is not prohibitive. Also, if the angular momentum in the gas around the star is high, a rotating cloud may be formed around the star; it is then possible and even likely that gas captured on a later occasion, with different angular momentum, will lead to collisional cancellation of the transverse motions and so all the gas will fall into the star.

This brings up another question that was considered a few days ago. The ordinary method in hydrodynamics is to start from low Mach numbers as the first approximation. In astronomical work we need high Mach numbers and it may be better to start from a cold gas (infinite Mach numbers) as a first approximation. In that respect our paper in 1944 (Bondi and Hoyle, M. N. **104**, 273, 1944) on the accretion of cold gas may be significant.

There is still one observation which I should like to bring forward in connection with a remark made by Biermann at the end of the discussion after McCrea's paper. The following points require consideration:

(i) It is clear that the luminosity of stars will impose a limit beyond which accretion becomes impossible. Qualitatively this agrees with observation, since there appears to be an upper limit to the luminosity (and hence mass) of stars. The only disagreement is on whether this observed limit is not higher than the limit that would seem to follow from Biermann's arguments.

(ii) The accretion theory of the origin of O- and B-stars does not say that these stars are accreting now, but that they accreted in recent times, when they were less massive. Consider an F- or A-star that begins to accrete heavily. The luminosity of such a star can clearly stop accretion neither by radiation pressure nor by heating. As the star grows its luminosity will increase, but the point at which it can succeed in stopping an already existing very dense accretion flow will be considerably higher than the luminosity level required to prevent the formation

of such a flow. A detailed calculation has not been carried out and would no doubt be difficult, but it seems likely to me that it would place the level of such a 'dynamic' limit into the region of the observed limit of luminosities.

MENZEL: Besides radiation pressure and angular momentum as possible factors affecting accretion, I should like to mention that magnetic fields may act as a deterrent to accretion. A magnetic star, encountering a cloud of ionized gas, will in effect repel the cloud from regions where the energy density of the stellar field exceeds the kinetic energy density of the gas cloud relative to the star.

HOYLE: Aren't there always places where the material can slip in along lines of force?

GOLD: It is necessary to make a distinction between the field of the star and the fields carried by the gas.

MENZEL: I mean to say that a field of the star, as for instance the sun's general magnetic field, is an unfavourable factor for accretion. I agree with Hoyle that in this case the material may come closer at the north and south poles, but there is no certainty that an ionized cloud can actually reach the stellar surface.

BONDI: Fields carried by the gas will increase by contraction (as shown by Batchelor on Wednesday), but not as strongly as the mechanical forces. So the power of magnetic fields to stop accretion will diminish as the material moves in.

BIERMANN: My objection to Bondi's simple explanation is that GM/r is not the proper formula for the potential. In the region in question there will be in general many later type stars which effectively determine the gravitational potential in most of the H II region around the early type star considered. As a source of ultraviolet radiation, that can accelerate interstellar matter, the sphere of action of such a star is much greater than that of its gravitational potential. This is quite different in the case of the initial contraction of a diffuse mass without a star-like condensation. As long as no star has been formed yet, we have to do only with infra-red radiation into which the mechanical energy is dissipated.

While thus holding a different view regarding the early type stars I would like to state, that I quite agree with Bondi's suggestion about the spectrum of T Tauri stars.

BONDI: Is my standpoint correct, if we call a star any region where the density greatly exceeds interstellar density?

BIERMANN: Of course it is vague to talk about the surface of a star

in these problems. But even if we consider gravitation only, I should not like to talk about a "star" larger than one parsec.

SCHATZMAN: I should like to make three more comments.

(1) During the fall of the matter toward the star, it is necessary to take into account all the laws of conservation: conservation of energy, conservation of momentum, conservation of matter. Bondi, in his paper has taken into account only the last one. In order to take into account the first two laws, it would be necessary to solve exactly all the radiation problems, including the radiation transfer problem.

(2) An enormous amount of energy is available. In the case of the sun, any atom falling on the sun would bring with it a total energy of 3.10^{-9} ergs, an energy which it is extremely difficult to get rid of. A tremendous elevation of temperature in the inner part of the nebula falling on the star is to be expected, which elevation, I believe, would prevent accretion to occur.

(3) Hayes asked about the mean free path. If the cloud moves with a velocity of 10 km/sec relative to the star, and has a temperature of 10^6 degrees, the diameter of the focal axis cannot be very small. The mean free path, as an order of magnitude is $\lambda \backsimeq 3.10^5\ T^2/N$. For the case under consideration: $T \backsimeq 10^6$, $N \sim 10^6$, $\lambda \backsimeq 3.10^{11} \backsimeq 4$ solar diameters. The mean free path itself is far from being negligible.

TAYLOR: I gather that the radial flow is considered to be supersonic. But as the material moves in it cannot penetrate the sphere where the velocity equals the velocity of sound.

BONDI: In the case considered, the gas is not driven by pressure gradients but falls in under the influence of gravitation.

FRENKIEL: I still do not know what the conclusion of this discussion is. Could we not start from the other end, by the well established fact that there are, apparently, clouds that are contracted into stars?

BONDI: Apart from the opinions of a very few people (e.g. Jordan) this is universally accepted.

ZANSTRA: The radiation pressure has been too lightly dealt with. A great many papers have been published on this subject, which is not simple by any means. It is not true that the momentum lost is equal to the force of radiation pressure, though it approximately holds for the Lyman continuous radiation. What is forgotten is that this Lyman continuous radiation is transformed into Lyman α radiation, which then is scattered. Compared with the continuous radiation the absorption coefficient is 10^4 times larger; so the radiation pressure would be 10^4 times larger, except for the effect of redistribution in the line profile due

PART VII

GAS AND DUST IN THE INTERSTELLAR MEDIUM

CHAPTER 30

ON THE MASS BALANCE OF THE INTERSTELLAR MEDIUM

BY

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The main processes influencing the mass balance of the interstellar medium are probably:

1. Rapid loss of gaseous shells or envelopes
   (a) by supernovae and novae
   (b) by planetary nebulae
   (c) by other stars (e.g. P Cygni stars);
2. Continuous corpuscular emission
   (a) by Wolf Rayet stars
   (b) by supergiant stars
   (c) by main sequence stars of about solar type
   (d) by single early type stars
   (e) shedding off of matter by fast rotating single or by close double stars;
3. Continuous accretion of interstellar matter
   (a) by sufficiently massive stars in ordinary interstellar clouds (leading eventually to rejuvenation)
   (b) by ordinary main sequence stars in very dense clouds (emission line stars);
4. Birth of stars in very cold and dense regions of interstellar material.

One major problem is the following: Is the interstellar material on the whole mainly secularly losing mass to the benefit of the stars, or has a considerable fraction of this material once or repeatedly been condensed into stars and re-emitted as corpuscular radiation into interstellar space?

Observations of the spiral arms, say in the Andromeda nebula, seem to indicate that the stars earlier than $\approx$ A 0 are being born and evolve

to some other type in a time comparable with that of one revolution around the centre of the nebula considered (some 10^8 yrs), which should indicate the order of the life time of any particular spiral arm.

Concerning the interaction of material falling into stars and of corpuscular emission by the same stars (processes 2 and 3) it must be emphasized (as has been mentioned already) that most probably they cannot both operate at the same time for a particular star; this is so on account of the very strong interaction especially of the ionized constituents and the sufficiently large cross sections for ionization by collision or by exchange of charge. Also the emitted material is likely to carry magnetic fields by which the interaction may be still further amplified.

It is estimated, that in our galaxy the following amounts may be involved (for an independent earlier estimate see A. D. Thackeray: On Some Possible Evolutionary Trends in the Interstellar Medium [1]).

1. Rapid loss of gaseous shells or envelopes

(a) Supernovae: $1\, M_\odot/100$a . Novae: $10^{-5}\, M_\odot \cdot 10^2$/a. ($M_\odot = 2.10^{33}$ gr).

(b) Planetary nebulae: estimated number 500, estimated average loss $M_\odot/30000$ a. $\rightarrow$ 1 $M_\odot/60$ a. (cf. Wurm: Die Planetarischen Nebel, Berlin 1951).

(c) A quantitative estimate is difficult, but it seems unlikely, that these stars contribute more than 1 (a) or (b).

Summing up, it appears that processes 1 (a) (b) (c) can together bring hardly more than $\approx 0{,}1\, M_\odot$/a. or $3.10^8\, M_\odot$ in 3.10^9 a. into interstellar space. On the other hand, the mass of the interstellar material is probably of the order of some $10^{10}\, M_\odot$.

2. Continuous corpuscular emission

(a) Wolf Rayet stars ($R \approx 6\, R_\odot$,[2] $L \approx 20\,000\, L_\odot$, $M \approx 12\, M_\odot$): Estimated loss 10^{20} atoms/cm²sec $\times$ 10^{24} cm^2 $= 10^{20,5}$ g/sec $= 5\, M_\odot$ in 10^6a. (:), estimated number 10^3 or $10^4 \rightarrow$ some $10^{-2}\, M_\odot$/a.

(b) Supergiant stars especially of type earlier than A. 1 $M_\odot/10^4$ a. per star is probably an overestimate; estimated number $\approx 10^4$ or 10^5 (the giants are not thought to lose much of their mass).

(c) Main sequence stars similar to our sun in their (low) activity: $M_\odot/10^{12}$ a. by corpuscular radiation. This estimate is based on the evidence from geomagnetism, cometary physics, observations of the corona and the zodiacal light. There are indications, however, that later type main sequence stars are often much more active than our sun.

(d) Around the early B-stars circumstellar absorption lines showing velocities of –20 or –30 km/sec are often observed. These are perhaps most easily explained as products of corpuscular radiation (if this is also emitted with velocities of the order of 1000 km/sec, it will accelerate a much larger mass of interstellar material around the star to low velocities). An argument of quite a different character has been proposed by Fessenkoff: If the mass of the early B-stars would remain constant, one should expect to find at least some stars in the final stage of almost complete consumption of hydrogen, that is with a much higher luminosity than would correspond to their mass. On the other hand, the probability to find such a star is rather low, since they can be only for a comparatively short time in this stage of evolution.

The corpuscular radiation, as shown by the sun, may be regarded as a special form of the non-thermal energy output, other such forms being the excess radiation in the far ultraviolet (particularly the Lyman and the near Röntgen region), that in the radio frequency range, and the acceleration of charged particles to cosmic ray energies. It seems, that of these the corpuscular emission in the velocity range of say 300–3000 km/sec, gives on the average the largest contribution to the total non-thermal energy output. All these emissions probably have their ultimate cause in the convection of certain subphotospheric layers and are highly variable, following partly the general cycle of solar activity; but that needs not concern us here.

Looking at the corpuscular emission from this angle, it does seem to be a natural first approximation to assume that it constitutes for all main sequence stars the same fraction of their respective luminosity L. If furthermore the kinetic energy per gram is approximately the same for all these stars, we would have $\partial M/\partial t \sim L$. An evolution scheme (discussed by Fessenkoff, Massewich and others) would result in which the early type B-stars would move downwards along the main sequence into the region of the A- or F-stars in times of the order of 10^9 a. (instead of moving approximately upwards in case of constant mass). The cosmogonical side of this problem cannot be discussed here.

If we have numerically a kinetic + potential energy of 10^{16} erg/g, we find with the assumption of an effective energy output of $L/100$ in form of corpuscular radiation $\partial(M/M_\odot)/\partial t \approx \frac{L/100}{L_\odot} \cdot 10^{-8,2}$

(b) *Ordinary main sequence stars in very dense clouds*

It seems difficult to arrive at a quantitative estimate of this effect. Perhaps as a guess it may be suggested, that its contribution might be at most of a similar order of magnitude as that of 3 (a), but is probably smaller on account of the slower collection rate to be expected for stars of this mass.

4. Birth of stars in very cold and dense regions of interstellar materal

The arguments presented in another connection seem to lead to the conclusion, that this process instead of 3 (a) should be believed to keep stationary in time the total number of early type stars. The quantitative estimates given before are not altered by this.

Summing up 3 and 4, it appears, that $\approx 1\ M_{\odot}$ a. should be a conservative estimate for the order of magnitude of the rate, at which interstellar matter condenses into stars or is being accreted (3 b). If a higher amount of corpuscular radiation is to be balanced in addition, the condensing rate must be correspondingly higher.

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Discussion

Bondi: Fessenkoff has put forward a powerful argument to show that luminous stars are emitting corpuscular radiation. According to the theory of stellar structure, the luminosity of a *homogeneous* star of given mass increases rapidly with increasing mean molecular weight. Since massive stars convert their hydrogen rapidly into helium, we should be able to see massive stars of widely differing composition and hence widely differing luminosities. As the observed mass-luminosity diagram does not show such a spread of luminosities, Fessenkoff concludes that there must be a process in operation that does not allow a star to remain massive for sufficiently long to permit it to change its composition appreciably. This process he takes to be loss of mass by corpuscular radiation.

To me this appears to be the only good argument put forward in

favour of corpuscular emission, but I do not think it is really firm. In the first instance the observed masses are quite uncertain, and secondly Mestel has recently shown that the mixing of these stars is very unlikely to be complete. If the star is not well mixed, it will not remain homogeneous, and no striking increase in luminosity will result from the production of He in the core. Accordingly I am far from convinced that corpuscular radiation forms an important process in stellar evolution.

One other point concerns the balance of matter between interstellar space and stars. If the stars lose a great deal of matter by corpuscular radiation, greatly enlarged accretion is required to keep up the balance. I doubt whether the requisite accretion rates occur sufficiently frequently.

Oort: The idea put forward by Biermann is that not the individual bright stars are kept constant by accretion, but that their number is kept constant by the birth of new stars.

Bondi: I agree that the very bright stars shed matter into interstellar space, either as routine or only at certain times and then explosively. I favour the second possibility.

Savedoff: There even seems a possibility that the O 5 stars of highest luminosity will lose sufficient mass in a century to make the effects on the apparent magnitude observable.

Blaauw: We should not forget that also some late type stars, e.g. the W Ursae Majoris stars, which are very frequent, supply a large fraction of the interstellar gas (point 2(e) in Biermann's Table).

Gold: It seems clear now, that matter oscillates between bright stars and interstellar space, but it is impossible to tell whether this is true also for small stars.

Schatzman: How is the situation in elliptical nebulae, where no interstellar gas is observed?

Biermann: Also the bright stars, that would furnish most of the gas, are absent there.

Savedoff: Have any flare stars of high velocity been observed?

Oort: No, there even is an opposite effect. The Me dwarfs, which are intimately connected with flare stars, are found to have systematically low velocities, comparable to those of B-stars. This has been found independently by Delhaye and Baade. It indicates that also these stars are quite young.

Hoyle: The loss of mass postulated by Biermann should become clear in changes of the orbits of double stars.

Oort: This effect would be difficult to distinguish from changes due to other causes.

CHAPTER 40

THE RELATION BETWEEN DUST AND GAS IN INTERSTELLAR CLOUDS

BY

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I propose to consider the frictional interaction of gas and dust and the effects of light pressure on their relative motion. Although no precise numerical conclusions can be drawn, an enrichment of the relative dust density by a factor of two by the processes contemplated would seem unlikely. One must be careful in comparing this conclusion with observational data on dust and gas, since it does not consider the destruction and growth of the dust in its position. It implies that one should not expect large enrichments of the elements bound in the dust relative to the hydrogen and helium of the gas. These conclusions are based on minor improvements of the pertinent developments of Spitzer [1] and Whipple [2].

We observe the reddening by the dust and the strength of the interstellar absorption and emission lines. A large number of processes affect this apparent relationship without contradicting the conclusion of this note. For example, the 21-cm hydrogen emission line may be weakened in some regions by molecule formation catalyzed by the dust surfaces. Absorption lines of Na I, Ca I, Ca II, etc., are affected strongly by the radiation field, curve of growth effects, and will be further weakened if an unusually large fraction of these atoms are frozen in the dust. Failure to observe H α emission from directions where O-stars and dust are present may be caused by separation in distance; however, a pure reflection nebula with an O-star source probably would be inexplicable if found. These are all mechanisms which we expect on the basis of our knowledge of the physics of radiation and matter. It seems, therefore, valuable to obtain an estimate of the interaction of gas and dust in the presence of light pressure and see to what conclusions one is led. This procedure is no substitute to a quantitative treatment of the processes mentioned above, but yields at least suggestive results.

The interaction between gas and dust can be divided into a purely electrostatic "dynamical friction" and the friction produced by physical collisions with gas particles. The following numerical data apply to an interstellar H I cloud with $n_H = 10$ and $T = 100°$, containing dust of 1/4 micron radius and density 1.3 gm/cm³. These particles are brought to rest from a velocity of 1 km/sec in 0.435 parsec. For velocities larger than 1.9 km/sec, the stopping distance increases to $0.29 + 1.93 \log V$ parsec. Hence for no reasonable initial velocity will the range of the particle exceed the diameter of a typical cloud. These particles will be reduced to a speed of 0.01 km/sec from any initial velocity in about 2.7×10^6 years. The distances and times are inversely proportional to the density; thus one may suppose that for denser regions one encounters even shorter ranges. Further, these numbers are upper limits, as they are computed with expressions which in the extreme underestimate the force by 40 %.

For charged grains an additional force is produced by "dynamical friction". Under the conditions outlined above, a charge of perhaps −0.03 volt is expected (about 5 to 6 electrons). When the kinetic energy of the atoms is lower than the mean interaction energy some mathematical difficulties with the approximations are encountered. For velocities below 1 km/sec, the charge-dependent friction is comparable to collisional friction. For higher velocities the force is proportional to v^{-2} and may be neglected. In H II regions a charge of −3 volt is expected, and the penetration distances and times are 10^{-4} to 10^{-6} of those in H I regions. Thus for most of the material in the universe, no separation of dust and gas can be anticipated in the absence of specific forces.

Radiation pressure is one force which acts to separate grains and gas. This force in the past has been overestimated through neglect of the high albedo or the reflectivity of the particles. As an example, we will try to calculate the velocity of a dust particle on the fringe of the "Coal Sack". A measurement of the surface brightness by Hoffmeister[3] yields $4^m.18$ per square degree, which is almost equal to the average sky brightness. To obtain the maximum effect of light pressure consistent with this observation we might consider the radiation field on the particle as made up of two parts: the Milky Way light idealized as a line source, half of which is blocked by the cloud; the light reflected by the cloud, represented by isotropic radiation from the direction of the cloud. Thus the diffuse light reflected by the cloud has the least possible flux, while the half ring has the most net flux, provided the essential features of the problem are preserved. The intensities are distributed in such a way that

the radiation density is unchanged. With this model the velocity of a particle is about 0.06 km/sec for $n_H = 10$ and proportionally smaller for higher densities. In 2×10^7 years the sphere of particles collapses 1.2 parsec. Even with the assumption of spherical symmetry this motion changes the mean dust density by 2.3. One cannot expect general concentrations in the interstellar medium to remain undisturbed over longer periods.

In cloud collisions it is true that the penetration of atoms is far smaller than the ordinary stopping distance of the grains at normal density. However, in the boundary layer formed during collisions, the temperature and ion densities will be much higher than normal, and the boundary layers of high gas and grain density should be comparable in size.

Upon this basis, the nonexistence of stars of low hydrogen and helium content, predicted on the basis of the collapse of dust clouds, is explained. It is improbable that any large portion of the interstellar dust is streaming through the gas at supersonic velocities. Lastly, regions of high dust density which are so noticeable on direct photographs are probably regions of high gas density (if they are large enough). These regions may originate in a compression of the general interstellar medium. It seems that many of these regions interact only for relatively short times with other regions and that a discrete region rather than a continuum-mechanics may be indicated.

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CHAPTER 41

REMARKS ON THE CONSTANCY OF THE RATIO BETWEEN GAS AND DUST IN INTERSTELLAR SPACE

BY

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1. The Problem

Current trends of thought favor the assumption that the ratio of the densities of interstellar gas and dust is about the same in all parts of our Galaxy. Theoretical arguments—like those presented by Savedoff in the preceding note and earlier by Spitzer [1]—show that the "stopping distance" of an erring dust particle in a medium of neutral or ionized hydrogen with a gas density $n_H \geq 10$ is relatively so small (of the order of one parsec or considerably less) that the dust is literally dragged along by the gas. It is then considered unlikely that any concentration of dust will be formed without a corresponding concentration of the gas, the ratio between gas and dust density being presumably close to a constant average value.

I grant—with some reservations—the general validity of these theoretical arguments. One should, however, keep in mind that a density $n_H = 10$ corresponds already to about 0.25 $\odot/\text{ps}^3$, which exceeds by a factor four or five the average probable gas density near the galactic plane. Also, while the theoretical arguments apply certainly to a relatively small section of our Galaxy, I can see no reason why this ratio should be the same, on theoretical grounds alone, for widely separated parts. In the end the decision should rest with the observations and, until conclusive evidence will have been found directly, workers in the field will do well to keep an open mind on the question of the constancy of the ratio gas to dust. There is at present no observational evidence to support the hypothesis of the constancy of this ratio and the limited evidence that is available seems to show that no proportional increase in the density of interstellar gas is found inside the densest concentrations of interstellar dust.

2. Relation of Cosmic Dust to Spiral Structure

We may start by recapitulating briefly the evidence for the relative distribution of gas and dust in galaxies outside our own. Baade and his associates have reported that the spiral arms of the Andromeda Nebula are dense concentrations of gas and dust as well as of O and B stars and Cepheid variables[2]. When one examines in detail the distribution of the dust, one finds strong indications that dust, while present throughout the spiral arms, is markedly concentrated toward the inner parts of the arms, where there is relatively little evidence for the presence of gas. Features like the Great Rift of our Milky Way from Cygnus to Sagittarius and Scorpius and the Southern Coal Sack—dark nebulae located at the inner part of Morgan's Northern Spiral Arm—remind one very much of some of the conspicuous features in the nearer spirals; some dissimilarity in the relative distribution of gas and dust is indicated. In the spirals dust does not appear to be a perfect "tracer" for interstellar gas.

3. Observational Evidence for our Galaxy

In our own Galaxy there is no clear-cut correlation between the intensities of the interstellar K and D lines and the color excesses of the relevant stars, other than the obvious correlation produced by distance effect [3]. Most important is the observation reported by Greenstein and Struve [4], who observed a star in the dark nebula in Ophiuchus which is reddened to such an extent that the total absorption must be about 4 mags and which yet shows only a very weak interstellar K line. While it is true that in specific cases one may interpret the lack of increase in intensity of the interstellar lines as caused by curve of growth or saturation effects, it does seem peculiar that in each case some special process had to be called upon to explain the apparent lack of dependence of the intensity of the interstellar lines upon the concentration of dust. Another case in point is the observation reported by Oort and van de Hulst in the present symposium according to which the observed intensity of the 21-cm line of H I is smaller rather than greater in the densest parts of the Taurus dark nebulae complex when a comparison is made with the less obscured surroundings. The lack of expected increase of strength may be understood as the result of decreased temperature and formation of hydrogen molecules, but we note again that a special process had to be called upon to explain the lack of direct evidence for

an increase in the gas density proportional to the observed increase in concentration of the dust.

Along the band of the Milky Way, there is evidence for a quite different relative distribution of gas and dust from one section to another. In the Carina section of the Milky Way, gas and dust are both plentiful, but in Sagittarius-Scorpius one finds for $l = 320°$ to $330°$, $b = -3°$ to $-7°$, plentiful dust and a fair number of early B stars and yet no evidence whatsoever for emission nebulosity [5]. It hardly seems as though the network of dark nebulae at $b = -3°$ to $-7°$ in Sagittarius-Scorpius is basically an intricate network of dense gas clouds.

We may mention at this point the result of a recent study by Campbell Wade and myself of the distribution in the southern Milky Way of O to B 2 stars and associated Hα emission. As an average for the entire southern Milky Way, we find for the O stars associated emission nebulosity in 56% of all cases "certainly" and in another 20% of all cases "possibly". The corresponding average percentages are 20% and 14% for the B 0 stars and 17% and 23% for the B 1–B 2 stars. In some sections—like that in Carina— the percentage of O to B 2 stars with associated Hα emission runs higher than 80%. But in others the same percentage falls well below the average. The most remarkable region in this respect is that between $l = 270°$ and $280°$ (directly east of the Coal Sack), where we find a marked concentration of O to B 2 stars, considerable evidence for the presence of interstellar dust, but practically no associated emission nebulosity. In our first survey of the region east of the Coal Sack, Wade and I suspected the large diffuse nebula near $l = 274°$ to be an Hα emission region, but Gum pointed out that it was really a reflection nebula; this has since been confirmed by photographs made at Boyden Station.

It need not surprise us that *excessive* cosmic dust is not found near the diffuse nebulae like the Orion Nebula, Messier 8, the Carina Nebula and others with densities one thousand times or more the average for our galactic spiral arms. The radiation pressure from the associated O and B stars exerts sufficient radiation pressure to repel the dust particles if they should occur or be formed in the central parts of these gaseous nebulae. There seems to be general agreement among those favoring a constant average ratio gas to dust that this constancy does not apply to the H II regions near O and B stars. On the other hand, the proponents of the constancy of the ratio gas to dust suggest that it does apply to the dense regions of cosmic dust. If this were so, one is tempted to ask why no formation of O and B stars seems to be taking place in some

of the complexes of dark nebulosity like Taurus and Ophiuchus, where one would seem to have ideal conditions for star formation.

4. Calculated rate of growth of the Coal Sack

Some years ago, I investigated the probable rate of growth of dark nebulae by the falling in of particles toward the obscured cloud and by accretion [6]. The rate of increase of mass of typical small globules was found to be very small, but for the Coal Sack the rate of growth was rather large. Savedoff has repeated these calculations taking into account the high albedo of the particles and the radiation pressure from the Coal Sack itself. For an assumed density $n_H = 10$ for the gaseous medium surrounding the Coal Sack, Savedoff finds that the velocity with which the particles approach the Coal Sack is of the order of 0.008 km/sec. My own calculations referred to a gaseous medium with $n_H = 1.3$ and since the ultimate velocity of approach for the particles is inversely proportional to the gas density, we should expect $v = 0.06$ km/sec for $n_H = 1.3$, a value substantially in agreement with that found in 1947. Since 1 km/sec is equivalent to 1 ps/10^6 yrs, we may expect the Coal Sack to gain in 10^8 years the mass of the cosmic dust contained in a spherical shell with an inner radius equal to that of the Coal Sack (4 ps) and an outer radius of 10 ps. For an assumed average density of the cosmic dust in this sphere of 6×10^{-25} g/cm^3 = 0.01 $\odot$/ps^3, the total gain in cosmic dust for the Coal Sack in 10^8 years would amount to between 30 and 40 $\odot$. If we assume that the Coal Sack represents primarily a dust concentration, its present mass is probably of the order of 100 $\odot$ and the increase in mass by the falling in of field particles would then entail a marked increase in the total mass in the course of one galactic revolution.

5. A plea for open-mindedness

The present note will have served its purpose if it is considered as a plea for open-mindedness on the issue of the constancy of the ratio of gas to dust in interstellar space. We note first of all that whereas gas and dust may go together in fairly constant proportion in fairly large volumes of galactic space, there is no theoretical reason why this ratio should be more or less the same everywhere near the galactic plane; the observational evidence available to date seems to point to considerable local variation of this ratio. We should further bear in mind that the mobility of the dust particles is very much greater in a gaseous medium with low

values of n_H (1 to 2, corresponding to the average for the region near the sun) than for $n_H = 10$, the value frequently given as representative for the gas clouds. For the present it would seem desirable to consider in all theoretical work side by side two separate models, one in which the ratio of gas to dust is considered constant for all concentrations in the galactic medium and another in which the dust concentrations are considered as units composed principally of dust and imbedded in a general galactic gaseous medium with $n_H = 1$ or 2. It would be a pity if, in our eagerness to obtain a simplified model, we would neglect to examine the probable evolutionary status of clouds primarily composed of cosmic dust.

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[4] See J. L. Greenstein's Chapter, Hynek, "Astrophysics", p. 570–571, McGraw-Hill (1951).

[5] B. J. Bok, "Vistas in Astronomy" (in press).

[6] B. J. Bok, Harvard Observatory Monograph No. 7, p. 71 (1948).

Discussion

Savedoff: I firmly believe that observations are needed to check any theoretical results. But I cannot at present accept any evidence for large differences in the ratio of gas to dust, for:

(1) The curve of growth effects are quite serious in dense clouds.

(2) If Bok's suggestion that there are large concentrations of dust with little gas is correct, there should be reflection nebulae connected with O and B stars. Have they ever been observed?

(3) The evidence that gas is more concentrated to the plane of the Galaxy than dust is very weak.

Minkowski: I wish to underline this last statement. E.g. the region around the north pole is known to have reddening, but it also is full of faint emission nebulosity.

Schatzman: Four years ago I made a theoretical investigation and concluded that only towards the end of the evolution, i.e. when clouds

have contracted into globulus, the ratio of dust particles to gas may increase (Ann. d'Astrophysique **12**, 161, 1949).

BOK: So far I have heard no theoretical argument to prove the ratio should be the same from one region to the other.

OORT: The particle sizes are similar; that is one argument.

BOK: To Savedoff's second point I may mention a very large reflection nebula near the Southern Coal Sack, which occurs in a region with plentiful O and B stars. There are whole sections of the Southern Milky Way in which O and B stars prevail, for which there is good evidence for interstellar solid matter, but for which Hα emission is either absent or very weak. In this same connection, I should remind you that dust concentrations are often found in spiral galaxies (at the inside of spiral arms, for example) at places where gas is certainly not present in emission.

OORT: I think Baade would not accept this as a definite result.

BOK: Again: everybody should feel very uneasy on the basis of the data now available and should keep an open mind to possible changes in the ratio.

SEATON: It is possible that dust grains play an important role in interstellar chemistry. What is the correlation between reddening and interstellar molecular lines?

MINKOWSKI: Very little is known about the molecules. But there are the 4430 band, strongly correlated, and the red Merrill bands, less strongly correlated with reddening. Both are unidentified. This is a very important open question.

DEUTSCH: The early estimates of 95% H I regions and 5% H II regions are often quoted. They were based on the hypothesis of no genetic relation between O B stars and gas clouds. Since then the study of associations of O and B stars started by Ambartsumian has shown that the relation between these stars and gas is not fortuitous. Does that not invalidate the early estimates?

BOK: The observational evidence regarding percentage distribution of H I and H II regions is very much in a state of flux. We had better postpone attempting anything resembling a final conclusion until the current Hα surveys will have been completed and until new data will have been obtained with regard to average emission measures for the various observed features. Analysis of regional surveys of observed intensities of the 21-cm radiation of H I should contribute further to the answer to Deutsch's question.

OORT: The whole 10 percent and 10 particles per cm^3 business is a

poor approximation to reality, not much better than a guess. The distribution is very irregular. I like to warn against sticking to this picture too closely. But it has been useful and may still prove useful.

BONDI (chairman): We shall have to leave the rest of the vacuum-cleaner problem for private discussion.

PART VIII
CONCLUSIONS

CHAPTER 42

SUMMARY OF THE AERODYNAMICAL ASPECTS OF THE SYMPOSIUM [1)]

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The following attempt to summarize some of the results of the Symposium is mainly directed at the aerodynamical aspects of the subject, in a similar way as was done in the Summary of the first Symposium at Paris in 1949. An excuse for choosing such a point of view may be found in the circumstance that aerodynamicists are somewhat in the minority at this meeting, so that one need not be afraid that the astrophysical aspects will be driven too far into the background of the minds of the attendance.

For aerodynamicists it is of great importance to have learned that the picture of gas flow under adiabatic conditions does not always form a satisfactory starting point for studying important forms of motion of the interstellar gas. The interstellar gas, considered energetically, is not "self-contained", but finds itself between powerful sources of energy, formed by high-temperature stars, and a sink, represented by the almost empty intergalactic space. It is true that adiabatic conditions may be obtained in particular circumstances, some of which have been mentioned in communications read before the Symposium, but the majority of the interesting cases are influenced by energy exchange.

Data concerning the sources of energy have been summarized by Schlüter and by Oort in their lectures of Thursday morning (Chs. 27 and 28). The primary source of energy must be found in nuclear reactions in young O and B stars; the energy becomes available in the form of radiation of short wavelength. Estimates have been given of the total

1) This Summary has been written down in the present form after the Symposium. I am indebted to Dr. van de Hulst for having read the text and for advice on some points.

energy output by these sources and the direct influences of the ultraviolet radiation on the gas surrounding such stars (radiation pressure and ionization phenomena) have been considered. Further data have been presented by Savedoff in his contribution to the discussion following Oort's lecture (Ch. 29), and by Schatzman and Kahn in their paper on the motion of H I and H II zones (Ch. 30). Additional points of view were presented by other speakers taking part in the discussions.

Savedoff, in his communication of Tuesday morning (Ch. 11), has collected detailed data concerning the various ways in which the interstellar gas can take up energy through radiative and collisional processes, or loose energy through radiation of the gas into interstellar space. Supplementary information has been presented by Kahn (Ch. 12) and by Zanstra (Ch. 13).

It has come forward from these communications (as also from the original papers published by Spitzer and Savedoff on this subject), that the energy balance depends on a multitude of collisional phenomena, involving photons, atoms, ions, electrons, cosmic ray particles and dust, in which minute percentages of admixed atoms or molecules with low energy levels (or of dust particles) can play a decisive part. The consequence is that no simple expression can be given for the loss of energy in terms of average gas density and gas temperature; neither can the gain of energy be calculated in a simple way from gas density and the temperature and degree of dilution of the incoming radiation. For aerodynamicists this brings great difficulties. They will feel indebted therefore to Zanstra for his presentation of a simplified model of such processes. Even if it may be open to discussion if this model is applicable to the actual interstellar gas, its essential features are illuminating and can serve as an introduction to cases more closely approaching actual conditions. I will take the liberty briefly to recapitulate the basic equations of Zanstra's theory. In doing so I shall omit a few numerical coefficients of order unity, in order te bring out more clearly what is important from the aerodynamical side.

The gas will be assumed to consist of a single species of atoms, which can occur either in a neutral state A or in a singly ionized state A^+ (Zanstra's treatment is more general and gives attention to the possibility of multiple ionization). Let n be written for the total number of atoms per cm^3; $n_A = n(1 - \alpha)$ for the number of those in state A and $n_+ = n\alpha$ for those in state A^+, so that α is the degree of ionization. The ionization energy is Q. — It is further supposed that the atoms in state A have two energy levels with a relatively small energy difference χ. Let

there be n' atoms per cm³ in the state with lower energy and n'' atoms per cm³ in the state with higher energy, so that $n' + n'' = n_A$. — The number of electrons per cm³ in the case considered is equal to that of the ionized atoms: $n_\varepsilon = n_+ = n\,\alpha$.

When flow is absent, the energy equation for the gas at rest can be written:

$$\varrho\, \mathrm{d}E/\mathrm{d}t \;=\; N\,k\,(T_* - T) - F\,\chi. \tag{1}$$

Here E is the internal energy of the gas per unit mass; ϱ is the density in grams per cm³; N is the number of ionizations (changes from state A to state A⁺) produced by the ultraviolet radiation from the star, per cm³ per second; and F is the number of exciting collisions suffered by neutral atoms in the lower energy state.

As regards the ionizations each ionization after some time is followed by a recombination. In each cycle (ionization—recombination) the energy gained may be put equal to $k\,(T_* - T)$, omitting some numerical factors differing not much from unity (compare Zanstra's paper). In this formula T_* is the temperature of the star providing the radiation and T is the electron temperature, which is assumed to be equal to the gas temperature. The number N is equal to n_A multiplied by a complicated factor depending on the spectrum of the stellar radiation and on atomic constants; this factor is a function of the stellar temperature T_*, but does not depend on the gas temperature T.

Each exciting collision, to which the number F refers, stores the amount of energy χ in the atom. Afterwards this energy is radiated away into space and thus is lost. The quantity F is equal to the product of n' and n_ε, together with a factor depending on atomic constants and on T, in such a way that one can write:

$$F \backsim n'\, n_\varepsilon\, e^{-\chi/kT}\, T^{-1/2} \cong n_A\, n_\varepsilon\, e^{-\chi/kT}\, T^{-1/2},$$

if it is assumed that $n' \cong n_A$.

In a state of equilibrium: $\mathrm{d}E/\mathrm{d}t = 0$ and eq. (1) reduces to:

$$F\,\chi = N\,k\,(T_* - T). \tag{1a}$$

The number n_A drops out of this equation and there remains a relation determining the electron density, which can be brought into the form:

$$n_\varepsilon = n\,\alpha = C\,(T_* - T)\,T^{1/2}\,e^{\chi/kT}. \tag{2}$$

The coefficient C depends on atomic properties and on the magnitude of T_*. — In an equilibrium state there is the further relation:

$$n\,\alpha^2/(1 - \alpha) = e^{-Q/kT}\,T^{3/2}.\,(\text{factor depending on atomic constants}).$$

If a is near unity, [2]) one finds $1 - a \backsim n\, e^{Q/kT}\, T^{-3/2}$.

Since $\varrho = n\, m$ and $p = (1 + a)\, n\, k\, T$, it follows from (2) that:

$$\varrho = (m\, C/a)\, (T_* - T)\, T^{1/2}\, e^{\chi/kT}; \tag{3}$$

$$p = \{(1 + a)\, C/a\}\, k\, (T_* - T)\, T^{3/2}\, e^{\chi/kT}\, . \tag{4}$$

Equations (3) and (4) together constitute the two equations of state for the gas, equivalent to what are commonly given as the thermodynamic equation of state and the caloric equation. They refer to a given constant value of the stellar temperature T_*; each value of T_* gives a particular value of the coefficient C.

Zanstra has shown that there may correspond *three* values of the density to a single value of the pressure, provided the stellar temperature T_* exceeds a certain limit, depending on χ and on other factors. The intermediate value of the density is unstable, but the maximum and the minimum values can co-exist, in which case a state is obtained with part of the gas condensed relatively to the rest. Evidently this possibility of condensation will be of great influence on the behavior of the gas; it can be compared with the condensation which under laboratory conditions may occur in water vapor.

Coming now to the case of a gas in motion, the equations of gas dynamics must be used. For motion in one dimension the equation of energy takes the form:

$$\varrho \frac{\mathrm{d}}{\mathrm{d}t}\left(E + \frac{1}{2}\, u^2\right) = -\frac{\partial}{\partial x}\left\{(p - f)\, u\right\} - \frac{\partial q}{\partial x} + N\, k\, (T_* - T) - F\chi. \tag{5}$$

Here u is the gas velocity; f has been written for the "viscous stress", due to the deviation of the molecular velocity distribution from the normal Maxwellian form, and q is the heat flow connected with this deviation. The internal energy per unit mass E is given by:

$$E = \frac{1}{m}\left\{\frac{3}{2}\,(1 + a)\, k\, T + a\, Q + \frac{n''}{n}\,\chi\right\}, \tag{6}$$

provided it can be assumed that a temperature T can be defined everywhere in the gas.

[2]) In this case—a near unity—it follows that n_A will be proportional to n^2. Also N becomes proportional to n^2, while F becomes proportional to n^3. Attention to these relations was drawn by Dr. Seaton in the final discussion; his observations have been printed at the head of the discussion on condensation and temperature regulation in interstellar gas clouds (Ch. 15).

For stationary motion the time derivative d/dt assumes the form $u \cdot \partial/\partial x$. Since ϱu = constant in this case, one can integrate with respect to x (choosing a convenient zero point), which gives:

$$E + \frac{1}{2}u^2 + \frac{p-f}{\varrho} + \frac{q}{\varrho u} = \text{Const.} + \frac{1}{\varrho u}\int_0^x \mathrm{d}t \left\{Nk(T_* - T) - F\chi\right\}. \quad (7)$$

An example to which eq. (7) can be applied is the propagation of a single shock wave with constant speed and strength through a gas of uniform density and temperature. The equation of energy must be supplemented, of course, by the continuity condition ϱu = constant (already used before) and by the momentum equation. It is convenient to choose a coordinate system in which the shock wave is stationary, at or near the origin.

At large distances before and behind the shock front the equilibrium condition (1a) must be satisfied, so that eqs. (3) and (4) will be applicable. When the relation between p and the specific volume $\varphi = 1/\varrho$ for a given value of T_* is pictured in a diagram as was given by Zanstra, one can make use of the formula:

$$(p_2 - p_1)(\varphi_1 - \varphi_2) = (u_1 - u_2)^2, \quad (8)$$

which is an immediate consequence of the continuity and momentum equations. For a known initial state of the gas (p_1, φ_1) and an assumed velocity difference $u_1 - u_2$ this formula determines a hyperbolic relation between p_2 and φ_2. The point of intersection of the hyperbola with Zanstra's curve fixes the values of p_2 and φ_2. Since the decrease of the velocity implies an increase of pressure, the specific volume φ_2 will be smaller than φ_1 (condensation), and it also follows that the temperature T_2 of the gas in the condensed phase will be *lower* than the original temperature T_1. The latter feature is a direct consequence of the highly increased loss of energy by radiation determined by the greatly increased value of the number F; the low temperature leads to the great increase of the density.

The radiation term in equation (7) occurs in the integral with respect to x. This means that it will become of importance only at a certain distance from the shock front. There may be cases where this distance is large in comparison with other important distances in the field. In the immediate neighborhood of the shock front its influence can then be neglected, so that here one will obtain almost the same conditions as are predicted by the usual adiabatic theory, meaning that there will be a zone of very high temperature, much higher than T_1, just behind the

shock front. It will be an interesting problem to work out the distribution of temperature, density, pressure and intensity of radiation in the transition zone from this domain of high temperature towards the equilibrium state with low temperature and high density, which is obtained farther away. In particular it will be interesting to find the breadth of the region of strong emission of radiation.

We may leave the application of Zantra's theory at this point. It will have sufficed to make clear the importance of an analysis of the collision phenomena between photons and atoms or ions and electrons in the interstellar gas. It is of great value that these phenomena can now be attacked also from the experimental side in the laboratory. Judging by the communications presented before this Symposium by Kantrowitz (Ch. 16) and Schaaf (in the discussion on shock waves, Ch. 17), which have been supplemented by contributions from Laporte, it is evident that much information has become available which at the time of the Paris Symposium was not yet there.

It should not be left out of sight, however, that owing to the extremely small density of the interstellar gas and the very high velocities which may occur in it (sometimes of the order of several thousands of kilometers per second), there are many cases in which it is not allowed to assume that an equilibrium distribution of the energy over the various possible states is reached. (Attention to this point was drawn by Thomas in a remark made during the final discussion, Ch. 44).

Great attention has also been given at this meeting to the electromagnetic fields which are set up in gases in consequence of ionisation, owing to the circumstance that ions and electrons have widely differing free paths. Already at the Paris Symposium a paper was presented by Miss Helen Kluyver on the separation of charges which occurs in the collision of an expanding shell of gas with an interstellar cloud (Problems of Cosmical Aerodynamics, p. 89). A paper by Denisse and Rocard, published in the "Journal de Physique et le Radium", t. **12**, pp. 893–899, 1951, treated the excitation of electronic oscillations in a shock wave. At the present meeting the unequal acceleration of ions and electrons has been considered in the communication presented by Schlüter (Ch. 9), while a particular problem concerning the production of electric fields by the collision of two clouds was treated by Kahn (Ch. 20).

The general formulation of the equations for the electromagnetic field and its interaction with the motion of protons and electrons was presented in Schlüter's communication mentioned above, with important contributions in the discussion by Cowling and by others (Ch. 10).

The relations between the magnetic field and turbulent motions had already been considered at some length at the Paris Symposium. Renewed attention to these relations was given in Batchelor's communication on the asymptotic level which can be reached by the magnetic energy, and in his report on a paper by Chandrasekhar and Fermi, who were unable to attend the Symposium (Ch. 21). It is interesting to note that, according to the data given in the second part of that paper, the magnetic pressure at the outer boundary of a spiral arm of the Galactic system can be much larger than the gas pressure. — The problem of the maximum level of magnetic pressure has also been considered by Biermann (Ch. 25) and by Bullard (in a discussion remark to Ch. 21). The question has been raised whether there can be any "dynamo-effect", in the sense that the disturbance of the motion produced by a small magnetic field can lead to forms of motion which produce stronger and stronger fields.

There is, of course, also the influence of gravitation. The trend of thought at this Symposium has been that gravitation is a normal type of force, which has to be taken into account, but which does not lead to particular problems or to particular forms of instability. It plays an important part in the accretion problems considered on Friday (Chs. 36, 37 and 38) and in the problem of the origin of galaxies (Chs. 33 and 35).

Great attention has again been given to the problem of *turbulence*. Here a new point of view has come forward, differing from the main directions of thought at the Paris Symposium. It has become evident that the heating of the interstellar gas by hot stars is a very important source of turbulence in consequence of its random character both in space and in time. At the same time the difference between "compression turbulence" and "shear turbulence" has been brought to the foreground, a distinction which has been stressed by Liepmann in several discussions (see also the next chapter by Gold). Until now hydrodynamicists have been engaged mostly with turbulence of an incompressible fluid, in which rotation and deformation without change of density (*i.e.* vortex motion and shearing motions) are the only types of internal motion possible. We know that for certain boundary conditions, which in themselves need not present any random feature, the regular "laminar" type of motion of an incompressible fluid can become unstable and give way to irregular "turbulent" motion. This has led to the two great problems of the theory of incompressible flow: to account for instability; and to find the statistical pattern of fully developed turbulent motion as a function of a Reynolds number characterising the field.

When the medium is compressible, changes of density are possible along with vortex and shearing motions, and one can imagine a system of expansion and compression waves travelling through the medium in all directions, constituting a pattern of compression turbulence. It has been pointed out by Lighthill in his paper (Ch. 22) that shear turbulence is always accompanied by pressure fluctuations which act as a source of compression and expansion waves; hence shear turbulence in a compressible fluid will always be accompanied by compression turbulence. The intensity of the compression turbulence strongly increases with increasing Mach number of the flow.

In the interstellar gas the situation, however, seems to be different. The direct source of turbulence is to be found in the random character of the heating of the gas by stars (together with its accompanying effects as ionization and radiation pressure). Hence the stability problem need not be considered in this case and the primary form of turbulence is compressible turbulence. In consequence of the circumstance that the density is not a unique function of the pressure, vortex motion and shear turbulence now arise from the compression turbulence, so that the order of priority is inverted.

It is possible, of course, that the rotation of the Galactic system with its non-uniform angular velocity is another sourse of turbulence. But its importance has somewhat diminished. Moreover, the distribution of the angular velocity rather seems to make the rotation stable, so that it probably cannot furnish an important source of energy for turbulent motion.

It is of interest to give attention to the form compression turbulence may acquire. Owing to the non-linear character of the equations of motion for a gas, there will arise shock waves through distortion of the compression waves. As Kantrowitz mentioned in his lecture, this can be envisaged as a kind of "organising" effect: certain features present in the field of wave motion become enhanced to such a degree that they assume the character of individual entities, the development and history of which can be followed during a certain period. It will be an important, but at the same time a very difficult problem to construct a statistical treatment of such an assembly of shock waves, taking account of the interactions. This might be considered, perhaps, first for the case of one-dimensional motion, which case might give a clue to the energy transport by "acoustic waves", as is assumed to take place in the interior of certain stars. The actual problem for the interstellar gas refers to a threedimensional field, in which the interaction of shock waves can lead to the appearance of vortex sheets. The situation is complicated

by the extremely irregular distribution of the interstellar gas, while moreover the dissipative effects of viscosity and heat conduction, together with the effects of electric and magnetic forces may be of importance.

In this connection Biermann has remarked that the presence of shock waves as the main loci where dissipation of energy takes place, may have a bearing on the spectrum of turbulence (see Ch. 25). In shock waves there is a direct transfer of energy from large scale turbulence (scale of the order of the distance between successive shock waves) towards dissipation layers with a thickness of the order viscosity divided by the jump in normal velocity. This makes the total dissipation per unit area of a shock wave to be of the order of the third power of the velocity jump and independent of the viscosity (unless the magnitude of the viscosity is appreciably changed inside the dissipation layer); hence the average dissipation per unit volume of the field becomes of the order (velocity jump)^3/(mean distance between successive shocks). Although an analogous result is sometimes given for shear turbulence at high Reynolds numbers, the mechanism in that case appears to be different, since the thickness of transition layers exhibiting a difference of tangential velocity v_t tends to be of the order $\nu^{1/2}\, l^{1/2}\, v_t^{-1/2}$ (where l is the length of the layer in the direction of flow). It is possible that some kind of "cascade" mechanism plays a part here, ultimately leading to a more rapid dissipation; while also the longitudinal extension of vortex lines leads to a rapid dissipation. In any case, there is a subject for investigation here, the outcome of which is not yet clear.

I have made no comments thus far on the investigation of accretion problems, to which the discussions of Friday morning have been devoted, and to the theory of the acceleration of interstellar clouds, as brought forward by Oort (Ch. 28). Important as they are from the astrophysical point of view, I have the impression that from the aerodynamical side they look more as applications, stressing the necessity of studying the fundamental questions referred to before, but bringing no new particular aerodynamical riddles. It is beyond my power to summarize them at this moment, and the same applies to all the astronomical observational material brought before the Symposium. I would therefore conclude with mentioning a point which may directly concern the aerodynamicist: the interpretation of data on turbulence deduced from observations on irregularly distributed radial motions observed in certain gaseous nebulae. It seems that here a detailed investigation will be required concerning the meaning of the observational material.

Many more problems have been touched upon either in the papers presented before the Symposium, or in the many discussions. Undoubtedly every one who has been present will go home with his mind full of matters to think about. The hope may be expressed that new points of view will be reached within a short stretch of time, so that it will be desirable to organise a third Symposium of the same nature some years hence.

CHAPTER 43

CONCLUDING REMARKS ON TURBULENCE IN THE INTERSTELLAR GAS

BY

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Many things that came to my mind in trying to summarise my impressions of the discussions in the Symposium have been covered much better by Burgers' paper. I shall merely try to add some points from the side of astrophysics. During the past few years there has been much discussion about the ordinary theory of turbulence and the astrophysical conclusions that would follow from it. In that theory one is concerned with quantities like velocity distribution, vorticity, variations in pressure (but not in density). One thought that the major dynamical properties of the interstellar gas could be accounted for by that theory, though, of course, to do so might be complicated in detail. Also one thought that the interaction between turbulence and magnetic fields was fairly well understood, and that one would, therefore, be able to draw conclusions about the magnitude and structure of galactic magnetic fields.

All this time many astrophysicists were, however, very dissatisfied. The major observed effects in the galactic gas did not fit in with the theory of turbulence. There were huge variations in density; would they not invalidate alone most of the reasoning behind the theory of turbulence?

I felt pleased when I learned at this Symposium that a different type of theory will be required, for one can now hope that that new theory will then show a better correspondence with the observed features. The ordinary theory of turbulence is concerned with very low Mach numbers. The new theory that we shall require will apparently be concerned with high Mach numbers.

The occurrence of high Mach numbers can be more easily understood when we consider that such forces as gravitation, and not only pressure gradients, may be responsible for the motion. Lighthill's idea that

turbulence at high Mach numbers may be interpreted in terms of an assembly of shock waves seems to me an extremely fortunate one for explaining a variety of astronomical features. It has been pointed out by Kantrowitz that shock waves appear to make order from chaos; they can lead to the appearance of smooth shapes and contours which would fit the observations much better than the chaos of ordinary turbulence.

Various points must now be investigated afresh. For instance, can we expect the rate of energy dissipation in an assembly of shock waves to be slow enough for our purposes? In a shock wave a substantial part of the energy is transferred immediately into heat by compression and not, as in low Mach number turbulence, only through the slow agency of viscosity. If such heat can be radiated away, would the motion not decay extremely quickly? It may be that we are saved by the fact that those same shock waves will break up the medium into small clouds. When a shock wave is moving through a cloud it will tear off a certain part of it as there is no reflecting boundary, and that part will travel as a new cloud until it collides with another in which it will again set up a shock wave. My picture is, therefore, that the kinetic energy of mass motion goes into wave energy and then again for a large part into mass motion.

In the case of the galaxy as a whole we must, of course, include the gravitational potential energy in our considerations. It may be thought then that those shock waves that are moving perpendicular to the galactic plane will propel some clouds away from that plane. These will, in general, have much less than the velocity of escape and will therefore fall back again to contribute to the general pool of shock turbulence. In this picture the gravitation field provides the reflecting boundary which is essential for keeping the energy in. This seems to me the kind of model of a galaxy that the new theory would be likely to devise and that would fit the observed configurations and motions.

The question of the generation of magnetic fields would now have to be discussed afresh. In ordinary turbulence it is the shear motions that have been found to be of importance. In the new picture shear motions would seem to play a secondary role. There are large density fluctuations, and indeed the material is torn apart into individual pockets which may not do much rubbing on one another. One would guess that the high Mach number motion will be less effective in increasing magnetic fields. Equipartition of magnetic and kinetic energy seems unlikely in this case, and it may therefore be possible that the theory of the motion

could be approached successfully from the aerodynamic side alone. But this point will become clear when the aerodynamic theory of high Mach number turbulence is developed, and one will then be able to see how well it can be applied to the case of the galactic gas.

CHAPTER 44

FINAL GENERAL DISCUSSION

Chairman: Dr. H. BONDI

LIEPMANN: I wish to protect—for the time being—the shock waves from misuse. We should not suddenly expect everything from shock waves. For example, shock waves cannot be considered as a simple alternative to shear waves or turbulence. It does not seem possible to conceive an ensemble of shock waves of finite strength without a coupled shear field. The interaction between two strong shocks always creates a vortex layer and hence a random ensemble of shock waves will be coupled with a turbulent field of random vorticity.

FRENKIEL: I should like to make a request for more measurements to which the methods used by fluid dynamicists to describe the turbulence characteristics could be applied. A description based on more extensive data may prevent the misuse of the word turbulence. After all, turbulence is a statistical concept; we cannot describe the characteristics of a turbulent field using just a few clouds, but we need a large number of sample clouds whose nature is defined by using a statistical average. In the work reported by Courtès, some rather interesting results were obtained. I was wondering if it would not be possible to obtain more extensive data by measuring the simultaneous velocities at several points of the cloud using Doppler shift data by methods similar to those which Richardson and Schwarzschild have applied in the study of solar granulations (R. S. Richardson and M. Schwarzschild, Astrophysical Journal **111**, 351, 1950).

From the solar granulation data I have found with Schwarzschild some preliminary results concerning the spectrum of turbulence. We are, at present, analyzing the brightness distribution on the surface of the sun from which more complete results on another characteristic of solar turbulence can be obtained. However, from our preliminary results it appears that the spectrum of turbulent velocities presents a double hump. This shape of the spectrum may be explained by the existence of

two driving mechanisms responsible for the two maxima. Whatever is the cause of such two maxima in the turbulence spectrum of the sun, it may be a mild example of a phenomenon occurring in the atmosphere of the red giants (F. N. Frenkiel and M. Schwarzschild, Astrophysical Journal **116**, 422, 1952). I now wonder if one would not find a somewhat similar shape of the turbulence spectrum for the interstellar clouds which may lead to some conclusions along the lines of Biermann's paper. In any case I should like to emphasize the need for direct measurements from which the spectrum of turbulence of interstellar clouds could be obtained.

LIEPMANN: A double hump in the spectrum would not necessarily mean a double source of dissipation.

OORT: A great difficulty to the program proposed by Frenkiel is that the H II regions that emit these lines are not quiescent.

HOYLE: Another problem that should be settled by observations is the Chandrasekhar–Fermi theory of the magnetic field along a spiral arm. If the suggestion that the motion of clouds is only strong enough to cause a simple corrugation of a homogeneous field is correct, this should show up in the statistics of cloud motions. Or are they sufficiently random to tell that this is not the case? Is there a hope to find out?

OORT: There is a definite possibility by looking perpendicular to the arm and along it. So far, no evidence of a difference is available.

BATCHELOR: The "statistical assembly of shock waves" that has been mentioned in earlier sessions is not a very clear concept for me. We are trying to imagine the properties of a turbulent motion in which the Mach number is very high. A way of doing this which may throw some light on the situation is to think of the high Mach number as being produced by a very small velocity of sound. In the limit of zero velocity of sound, or infinite Mach number, hydrodynamic pressure loses its meaning and the various particles of the gas move independently of each other. Thus it becomes necessary to think in terms of particle dynamics rather than in terms of hydrodynamics. From this point of view the picture of discrete gas clouds moving independently in a rarified background medium and occasionally making collisions may not seem to be so strange.

HAYES: Batchelor's remark should be completed by adding another parameter, viz. the ratio of mean free path to some scale of the motion. If that is small compared with unity, small parcels will not behave as free particles.

With respect to the further discussion I should like to remind people

of the five blind men examining an elephant; the picture of physical reality certainly may be different when approached from astronomy or from aerodynamics. As important problems ensuing from our discussion I see the following:

1. Is $\sigma \nu$ large enough in clouds for spontaneous generation of magnetic fields?
2. Is magnetic energy equipartitioned with total turbulent energy or with small scale turbulent energy?
3. Is the energy source of luminous edges kinetic or radiational? Luminous edge classification?
4. Where does Menzel's "pinch" effect enter?
5. Are luminous filaments shock fronts or electric protuberances?

GOLD: In reply to Batchelor's comment, I would say that the finite velocity of sound may be an essential quantity for determining the scale of the motion. For example, the size of the gas clouds resulting from collisions that build them up, and shock waves that tear them apart, may well be determined by quantities involving the velocity of sound.

THOMAS: I would like to make two comments here, one on the Survey and Summary by Burgers and the other on a comment by Batchelor. First, Burgers has suggested that the caloric equation of state proposed by Zanstra might be that to be used to replace the more conventional adiabatic equation of state used in aerodynamics. I would like to point out that the chief difficulty with such an approach lies in the fact that it does not meet our main object here—namely, that of bringing together the two viewpoints of astrophysics and aerodynamics. The main problem here is to investigate the aerodynamic velocity field when radiation becomes important, and the astrophysical radiation field when a velocity field becomes important. In astrophysics, one is accustomed to making a type of local equilibrium assumption which relates the emission and absorption of radiation under the general assumption that energy tranfer is by radiation alone. Thus, this local thermodynamic equilibrium assumption describes a type of coupling among the internal degrees of freedom involving radiation processes only. In aerodynamics, one customarily makes a type of local thermodynamic equilibrium assumption which couples the macroscopic velocity field with the internal degrees of freedom of the gas that specify the internal energy of the gas. Radiation is not important. Thus, if one wants to consider a situation where both velocity field and radiation field are important, the

chief object of the equation of state considerations is to cover both these parameters. And, of course Zanstra's considerations do not involve any velocity field.

This situation simply highlights what is the main problem confronting the astrophysicist when he introduces a velocity field containing a non-trivial amount of energy—namely, what perturbation on the spectroscopic state of the gas is introduced by this velocity field. Clearly, a detailed investigation of the detailed microscopic processes—with the possibility of non-trivial departures from a thermodynamic equilibrium state—must be introduced. A rough cycle is: transfer of energy from the macroscopic velocity field into atomic kinetic temperature by elastic collision, transfer of energy from atomic kinetic temperature to electronic kinetic temperature by elastic collision, transfer of energy from electronic kinetic temperature to internal degrees of freedom of the atom by inelastic electron-atom collisions, radiation from the system. The most pressing problems in this connection are therefore twofold: (a) Evaluation of all the relevant inelastic collision cross sections, both electron–atom and atom–atom; (b) The development of methods of treating a gaseous ensemble not in thermodynamic equilibrium. We have attempted to make some progress in this direction by studying the statistically steady state of a gaseous atmosphere not in thermodynamic equilibrium and maintained by a mechanical supply of energy in addition to the radiation field on the one hand; and on the other, continuing the investigations mentioned earlier in the week on the meteor and ultraspeed particle problems. I would, however, emphasize again the necessity of this coupled microscopic–macroscopic approach, with a very critical look at any type of local thermodynamic equilibrium assumptions.

The remark on Batchelor's suggestion lies very much in the same vein. Batchelor has suggested that we may consider the high Mach number case by letting the velocity of sound approach zero rather than the velocity become very high. I would object strongly to this procedure, for the same reason mentioned earlier in the week—a difference in the energy level in the astrophysical and the aerodynamic conventional cases. If, for example, one starts with a gas at a fairly low temperature, say 300° under standard conditions, he may reach a rather high Mach number before it is necessary to consider a significant variation in γ. Thus, it does not make too much difference whether one has a low velocity of sound or a high gas velocity. On the other hand, if one has a free stream temperature of five or six thousand degrees, even a very small Mach number results in a significant change in γ—hence a con-

siderable difference in the way he will formulate a description of the gas dynamic problem over the case of the low temperature in the free stream situation. These attempts to represent high velocity aerodynamics by substituting high Mach number aerodynamics have also led us to considerable difficulty in the problem of heat transfer to meteors and artificial meteors.

SAVEDOFF: One question put by the hydrodynamicists to the astrophysicists is how to describe the interstellar gas in a reasonably accurate, yet simple way. Burgers has referred to the many complications. I suggest that one simplification may be allowed, viz. that the gas behaves as an ideal monatomic gas for rapid changes, if the density is between 1 and 10^6 atoms per cm^3. For lower n cosmic rays may become important, for larger n the cooling processes may saturate or the regions become opaque (which of course depends also on their size). Rapid changes means changes in a time small compared with the equipartition times t_e quoted by Spitzer and myself. Fast compressions that satisfy these conditions occur with $\gamma = 5/3$.

SCHLÜTER: (a) The importance of high-Mach-number aerodynamics should not be judged solely by the observed density differences, since density differences of a factor 100 are already present in a steady state where the interstellar gas is partly ionized by stellar radiation.

(b) With regard to magnetic fields Batchelor remarked on Wednesday that in the contraction of gaseous masses the growth of the magnetic energy density ($\backsim \varrho^{4/3}$) should be less important than the growth of kinetic energy ($\backsim \varrho^{5/3}$). From this it could be concluded that the interstellar magnetic fields (which are supposed to be originally in equilibrium with the kinetic energy of the interstellar gas) are of no importance to the accretion or to the birth of stars. But there are real difficulties. The ratio of stellar and interstellar densities is of the order 10^{24}, and from an interstellar field of only 10^{-6} gauss a stellar field of 10^{10} gauss would result. That gives a magnetic pressure which is by several powers of ten too large to be overcome by stellar gravitation. From this it follows, that the contraction must be highly anisotropic, even if the interstellar fields are rather small; the gas has to flow along the magnetic lines of force.

(c) On the other hand the magnetic fields may help us to get out of the difficulty with the angular momentum, since the magnetic field can transport angular momentum by the way of Maxwell's tensions. We are at Göttingen working on this very problem.

(d) If magnetic fields are important to accretion of stars, all early type stars should be highly magnetized; one would indeed expect stellar fields

of the order of millions of gauss on the surface of these stars. This seems to me to represent the most serious difficulty for the hypothesis of interstellar magnetic fields.

On the other side some additional support in favour of this hypothesis may be found in the indications for an interplanetary magnetic field of at least 10^{-6} gauss as derived from cosmic ray observations (cf. A. Schlüter, Z. Naturforschung **6a**, 613, 1951).

HOYLE: I have to defend the contraction hypothesis against Schlüter's remark. The conductivity in a globule *may* be sufficiently low to give effective ohmic dissipation.

OORT: I should like to comment on the problem of the angular momentum.

In general the interstellar material must contain far too much angular momentum to contract to a body of stellar dimensions. Even if we consider only the galactic rotation the average angular momentum in a cloud of density $n_H = 10$ cm^{-3} would be 10^5 times too high. We have roughly

$$m/m_\odot = 0.1\, r^3 n_H,$$

where r is expressed in parsecs. For a star of mass 10 $m_\odot$ we need a volume with a radius of 2 parsec, if $n_H = 10$. The angular velocity of galactic rotation being about 40 km/sec.kps, the average "rotational" velocity at the surface of a sphere with 2 parsec radius would be roughly 0.1 km/sec. Contraction to a B star with radius $5\, r_\odot$, or 1.5×10^{-8} parsec would then give a rotational velocity of 10^7 km/sec, which is of the order of 10^5 times more than the observed and permissible rotations.

A suggested solution is that good spots are selected. But it is a big factor and, in the expanding associations at any rate, the life time of the order of 10^6 years does not leave much time for such selection, nor it would seem, for magnetic braking.

It is suggested that a way out of this difficulty may be found in the effects of compression in colliding clouds. When there is sufficiently rapid cooling by radiation, as may be the case in dense clouds, compressions up to a factor of the order of 100 may well occur. There are observational indications of compressions to factors of even 10^4 in some nebulae.

Consider, first, the schematic case of two clouds in a head-on encounter. Suppose both clouds to have a rotational motion in the same sense (see Figure 1.). A compression layer will then be formed at the boundary between the two clouds. Suppose the density in this thin layer to be f times that in the original clouds. The angular momentum that

was present in the two clouds will cause the thin layer to rotate with an angular velocity of the same order as the angular velocity of the original clouds. In addition, the rotational velocities of the clouds will produce currents in the layer, in a direction parallel to its surface. If f is of the order of 100 the layer will be so thin that these currents will probably vanish in a time that is short compared to the duration of the cloud collision.

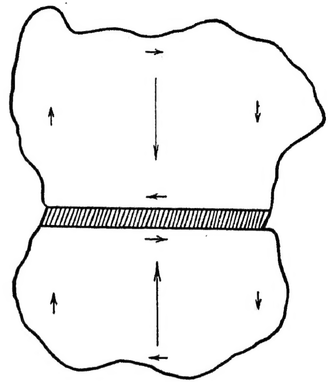

Fig. 1. Suggested reduction of angular momentum in a compression layer.

The radius of the volume containing enough mass to produce a star will be reduced by a factor $f^{1/3}$; the angular momentum will therefore be reduced by a factor of the order of $f^{2/3}$. A very considerable decrease of the angular momentum may thus be attained. In favourable cases the reduction may be still more. For if, instead of more or less regularly rotating clouds, we are dealing with clouds with irregular internal motions, these motions will be partly averaged out by the compression.

Gold: It depends, of course, on the speed at which angular momentum can be transported. The rough judgment obtained by comparison with the calculated case of such transport within the Sun suggests that long times would be involved.

Oort: It should be computed.